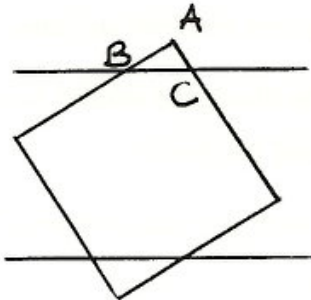
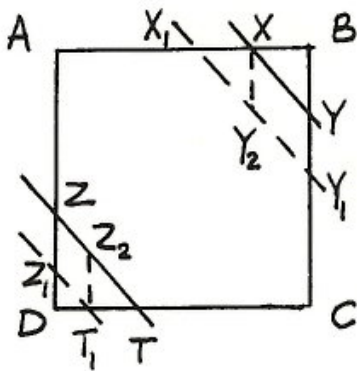


Esther Szekeres has provided us with a non-trigonometrical solution to **Question 866**. A trigonometrical solution appeared in the last issue of **Parabola**.



A 1 metre square masonry slab which formed part of a 1 metre wide path has become displaced as shown in the figure. A workman repairing the path saws off the two triangular pieces which project beyond the sides of the path. Find the sum of the perimeters of the two triangles.

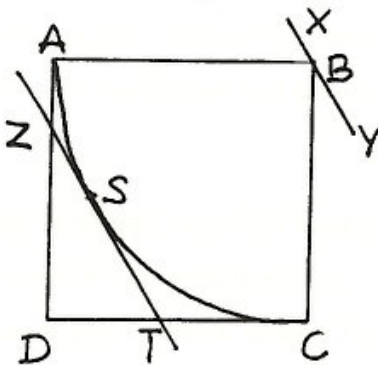
**Solution:** Consider the square which is intersected by two parallel lines, cutting out two triangles at opposite vertices.



We first prove that if the distance between the parallel lines is constant, then the sum of the perimeters of the two  $\Delta$ s is constant. Square  $ABCD$  has been intersected by the  $\parallel$  lines  $XY$  and  $ZT$ , cutting out  $\Delta$ s  $BXY$  and  $DZT$ . If both lines are moved such that the distance between them remains constant then the sum of the perimeters of the new  $\Delta BX_1Y_1 + \Delta DZ_1T_1$ , will be unchanged.

This is easily seen if we draw  $XY_2$ , resp.  $T_1Z_2$  as in figure,  $\parallel$  to  $BC$ . The two little  $\Delta$ s  $X_1XY_2$  and  $T_1TZ_2$  are congruent, and  $X_1XY_2$  shows the increase of the perimeter of  $BXY$ , while  $T_1TZ_2$  shows the decrease in the perimeter of  $\Delta DTZ$ .

In our problem the distance between the two parallel lines is 1 metre, the length of the side of the square.



Consider the special situation, where  $XY$  is passing through  $B$ . In that case  $ZT$ , having a distance of 1m away, will be tangent to the circle, centre  $B$ , passing through  $A$  and  $C$ . Perimeter of  $\Delta XBY = 0$ , so the sum is equal to perimeter of  $\Delta DZT$ . But  $ZS = AZ$ ,  $ST = TC$ , where  $S$  is the point of tangency, therefore

$$\text{Perimeter } \Delta ZTD = AZ + ZD + DT + TC = 2, \text{ independent of the direction of } ZT.$$