Parabola Volume 33, Issue 3 (1997)

PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q.1007 A student receives a mark out of 7 for each of the subjects English, Maths and Science. In how many ways can the student get

(a) a total mark of exactly 7;

(b) a total mark of at most 7;

(c) a total mark of exactly 16.

Q. 1008 If n is a positive integer, express n^3 as the sum of n consecutive odd numbers.

Q. 1009 If n is an integer, prove that the number $n^4 + 3n^2 + 3$ must have at least one prime factor of the form $4r + 3$.

Q. 1010 Take any right-angled triangle ABC, right-angled at A. Let T_1, T_2, T_3 be equilateral triangles drawn on the sides AB, AC and BC respectively. Prove that

$$\text{area of } T_3 = \text{area of } T_1 + \text{area of } T_2.$$

Q. 1011 The only solution to the equations

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 + x_3 &= 0 \\ x_3 + x_1 &= 0 \end{aligned}$$

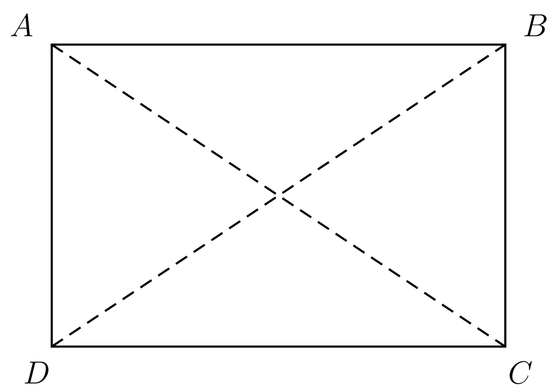
is $x_1 = x_2 = x_3 = 0$, but $x_1 = x_3 = 1, x_2 = x_4 = -1$ is a non-zero solution of

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 + x_3 &= 0 \\ x_3 + x_4 &= 0 \\ x_4 + x_1 &= 0. \end{aligned}$$

For what values of r and n do the following equations have a non-zero solution

$$\begin{aligned}
x_1 + x_2 + \cdots + x_r &= 0 \\
x_2 + x_3 + \cdots + x_{r+1r} &= 0 \\
&\cdots \\
x_{n-r} + x_{n-r+1} + \cdots + x_{n-1} &= 0 \\
x_{n-r+1} + x_{n-r+2} + \cdots + x_n &= 0 \\
x_{n-r+2} + x_{n-r+3} + \cdots + x_1 &= 0 \\
&\cdots \\
x_n + x_1 + \cdots + x_{r-1} &= 0.
\end{aligned}$$

Q.1012 The following figure shows that it is possible to join every pair of *four* points (no 3 in a straight line) with lines of 2 different colours without drawing a triangle of one of the colours:



What is the largest number of such points?

Q.1013 Is it possible for a knight to start at one corner of a chess board, visit every square of the board and end up at the diagonally opposite corner?

Q.1014 A sequence $p_1, p_2, \ldots$ is formed as follows:

$$\begin{aligned}
p_1 &= 2 \\
p_n &= \text{largest prime number which divides } p_1 p_2 \cdots p_{n-1} + 1
\end{aligned}$$

(For example, $p_2 = 2 + 1 = 3,\ p_3 = 2 \times 3 + 1 = 7$).
What is the smallest prime number which does not occur in this sequence?