Parabola Volume 38, Issue 1 (2002)

PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q1111 A grandfather clock takes 30 seconds to strike 6 o'clock. How long does it take to strike 12 o'clock?

Q1112 Simplify the following expression:

$$\frac{(2^3-1)(3^3-1)\dots(2002^3-1)}{(2^3+1)(3^3+1)\dots(2002^3+1)}.$$

Q1113 You read the following 5 graffiti on an otherwise blank wall:
Exactly 1 of these statements is false.
Exactly 2 of these statements are false.
Exactly 3 of these statements are false.
Exactly 4 of these statements are false.
Exactly 5 of these statements are false.

What can you make of this?

Q1114 What is the probability that a year chosen at random has 53 Sundays?

Q1115 If n is a positive integer, then it is known that $\dfrac{(2002n)!}{(n!)^{2002}}$ is an integer. Determine the highest power of 2002 that divides number.

Q1116 A game of draughts has to be abandoned as a draw even though the player to move has two kings and no other pieces while the opponent has only one king and no other pieces. In what positions could this happen?

Q1117 Prove that for positive real numbers a, b, c, d, e:

$$\frac{1}{a} + \frac{1}{b} \geq \frac{4}{a+b}, \qquad (1)$$

$$\frac{1}{a} + \frac{1}{b} + \frac{4}{c} + \frac{16}{d} + \frac{64}{e} \geq \frac{256}{a+b+c+d+e}. \qquad (2)$$

Q1118 Show that any polygon may be dissected into acute angled triangles.

Q1119 Let p, q be two positive integers with no factors in common (except 1) and $p < q$. It is desired to express p/q in the 'Egyptian fraction' form:

$$\frac{1}{n_1} + \frac{1}{n_2} + \ldots \frac{1}{n_r}$$

with $n_1 < n_2 < \ldots < n_r$ for some $r \geq 1$. Show that this is always possible.

Q1120 (a) Five identical solid cylinders whose diameters are 10 times their height are given. Show how to place them so that each touches each of the others. Can this be done with six such cylinders?

(b) Six identical solid cylinders whose heights are 10 times their diameter are given. Show how to place them so that each touches each of the others. Can this be done with seven such cylinders?