Parabola Volume 40, Issue 1 (2004)

Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*; your solution(s) may be used if they are received in time.

Q1151 Let $p(x) = (x^{2003} + x^{2002} - 1)^{2004}$. Find the sum of the coefficients of all odd degree terms in the expansion of the trinomial $p(x)$.

Q1152 Find the coefficients of x^{n-1} and x^{n-2} in the expansion of

$$p(x) = (x + \frac{1}{2})(x + \frac{1}{2^2})(x + \frac{1}{2^3}) \cdots (x + \frac{1}{2^n}).$$

Q1153 Find all values of a (real number) such that the system

$$x^3 - ay^3 = \frac{1}{2}(1+a)^2 \qquad (1)$$

$$x^3 + ax^2y + xy^2 = 1 \qquad (2)$$

has a solution, and that **all** solutions satisfy

$$x + y = 0. \qquad (3)$$

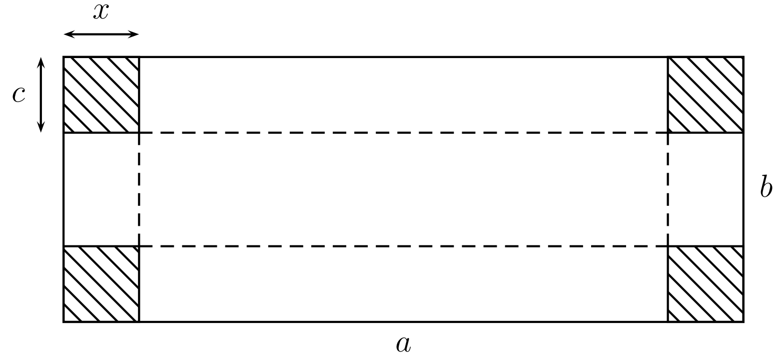
Q1154 Let a, b, and c be three positive numbers. Prove that

$$\frac{1}{a^2+bc} + \frac{1}{b^2+ca} + \frac{1}{c^2+ab} \leq \frac{a+b+c}{2abc} \qquad (1)$$

and

$$\frac{1}{a^3+b^3+abc} + \frac{1}{b^3+c^3+abc} + \frac{1}{c^3+a^3+abc} \leq \frac{1}{abc}. \qquad (2)$$

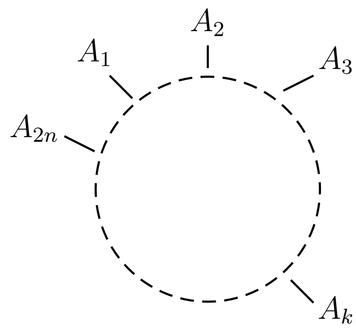
Q1155 Four equal squares of side x cm are cut from the corners of a sheet of metal of size a cm and b cm, as in the figure. We then fold the sheet along the dotted lines to make a container (without a lid). Find the value of x so that the container has a maximum volume capacity.



Q1156 Two friends Jack and Jane have to take a train at the same station to get to school. To ensure not to be late, they have to be at the station some time between 8am and 8:30am. They want to travel together on the train, but don't want to wait too long for each other. So they come to this agreement. Each one arriving at the station will wait at most 5 minutes for the other. What is the probability that they travel together?

Q1157 Let $A_1, A_2, \cdots, A_{2n}$ $(n \geq 2)$ be $2n$ points (in that order) on a circle such that $A_1 A_2 \cdots A_{2n}$ is a regular polygon.

Any 3 points form a triangle, while some set of 4 points form a rectangle. Given that the number of triangles is 20 times the number of rectangles, find n.



Q1158 Let $S = \frac{1}{1.2.3} + \frac{1}{4.5.6} + \cdots + \frac{1}{2002.2003.2004}$.
Prove that

$$S = \frac{2005 \times A}{1.2.3 \cdots 2002.2003.2004}$$

with some integer A.

Q1159 Prove that there exists a triangle ABC such that its angles are solutions of the equation

$$(56 - 65 \sin x)(80 - 64 \sin x - 65 \cos^2 x) = 0. \qquad (1)$$

Q1160 Jim has at the back of his house a piece of land of size $3^m \times 3^m$. He designs this land as in the figure, where he wants to pave the shaded area and grow lawn on the remainder.

Assume that $0 \leq a < b < c \leq 2^m$. Since he doesn't want to spend too much time to mow the lawn, he wants to maximise the paved area. Find the value of a, b and c such that the paved area is a f maximum. What is this maximal area?