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Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*; your solution(s) may be used if they are received in time.

Q1161 Find all values of x (real number) satisfying

$$\begin{aligned} &\frac{x-1}{2004}+\frac{x-3}{2002}+\frac{x-5}{2000}+\cdots+\frac{x-2003}{2} \\ =&\frac{x-2}{2003}+\frac{x-4}{2001}+\frac{x-6}{1999}+\cdots+\frac{x-2004}{1} \end{aligned}$$

Q1162 Two women begin to walk at sunrise, one directly from point A to point B, the other directly from point B to point A. They pass exactly at noon. The first reaches point B at 4pm, the second reaches point A at 9pm. At what time was sunrise that day?

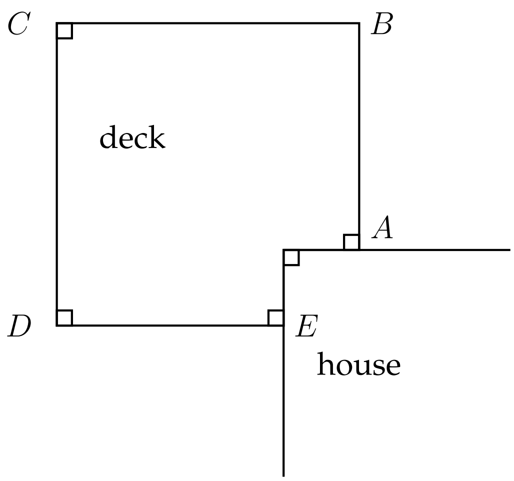
Q1163 One hundred cows each coloured black or white or brown stand in a field eating 100 bales of hay. Each black cow eats 5 bales, each white cow eats 3 bales, while it takes 3 brown cows to consume 1 bale of hay. Assume that all 100 bales are consumed and that there is at least one cow of each colour. How many cows of each colour are there?

Q1164 Among all pairs of positive integers (p, q) such that $p+q=2004$, which pair yields the maximum value $p!q!$ and which pair yields the minimum value $p!q!$? (Recall that for any positive integer n

$$\begin{aligned} &n! = 1.2.3\cdots(n-1).n. \\ \text{E.g.}\quad &4! = 1.2.3.4 = 24.) \end{aligned}$$

Q1165 Let n be a natural number and k be the number of distinct primes that divide n. Prove that

$$n \geq 2^k.$$



Q1166 Bill wants to build a deck at the corner of his house, as in the figure, where $AB = DE$ and $BC = CD$. He puts a railing around the outer edges of the deck. Railings are sold in length $6m$ each, and he buys two of them, intending to cut each into two pieces to have the four required railings.

How should he cut to maximise the area of the deck?

Q1167 The lengths of the sides of a triangle form an arithmetic progression with difference $\sqrt{2}$. Assume that the area of the triangle is 12. Prove that it is a right angled triangle.

Q1168 Inequality (2) in Question 1154 (Vol 40, No.1, 2004) can be generalised to

$$\frac{1}{a^4+b^4+c^4+abcd}+\frac{1}{b^4+c^4+d^4+abcd}+\frac{1}{c^4+d^4+a^4+abcd}+\frac{1}{d^4+a^4+b^4+abcd}\le\frac{1}{abcd}.$$

Prove this inequality.

Q1169 Given $T_1 = 1$, we define

$$T_{n+1} = 1 + T_1T_2T_3\cdots T_n \text{ for } n \geq 1.$$

(a) Prove that T_m and T_n are relatively prime integers if $m \neq n$.

(b) Prove that $T_{n+1} = T_n^2 - T_n + 1$ for $n > 1$.

Q1170 Let T_n be defined as in Question 1169.

(a) Prove that

$$\frac{1}{T_1}+\frac{1}{T_2}+\cdots+\frac{1}{T_N} = 2-\frac{1}{T_{N+1}-1} \quad \text{for all} \quad N \geq 1.$$

(b) (For Senior students.) Find the sum of the series

$$\sum_{n=1}^{\infty}\frac{1}{T_n}.$$