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Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*, and if received in time your solution(s) may be used.

Q1181. Consider the following set of linear equations

$$\begin{aligned} x + 2y + z &= 1 \\ -2x + \lambda y - 2z &= -2 \\ 2x + 6y + 2\lambda z &= 3 \end{aligned}$$

where x_1, x_2, x_3 are variables and λ is a parameter. Find all values of λ for which these equations do not have a unique solution.

Q1182. A homicide victim was found in a room that is kept at a constant temperature of $21°C$. A body temperature measurement was made at time τ and another was made one hour later. The results were:

$$T(\tau) = 27°C \quad \text{and} \quad T(\tau + 1) = 25°C$$

where time is measured in hours. Assuming that the victim's temperature was $37°C$ just before death, determine the time of death relative to time point τ. (You may wish to consider Newton's Law of Cooling, in which experiments show that the time rate of change of the temperature of an object is proportional to the difference between its temperature and the temperature of the surrounding medium).

Q1183. A bus takes 30 minutes to travel (non-stop) from terminal A to terminal B. Buses leave terminal A every 2 minutes, and travel at the same speed. A car leaves terminal A simultaneously with one of the buses, and travels to terminal B at 4 times the speed of the bus. How many buses will the car overtake by the time it reaches B?

Q1184. Find a polynomial of least degree and integer coefficients that has $1 + \sqrt[3]{3}$ as a root.

Q1185. Find all real roots of the following simultaneous equations

$$x^5 - y^5 = 2101 \tag{1}$$
$$x - y = 1. \tag{2}$$

This problem and its answer (to appear in the next issue) was suggested by Julius Guest.

Q1186. Find all values of m such that the equation

$$\cos^4 t + m \sin^2 t + 2 = 0$$

has a solution.

Q1187. At 12 o'clock the two hands of an analogue clock are together. At time t later they make an angle $\theta(t)$ (take this to be the smaller angle). How long after 12 are the two hands first in line and first together again?

Q1188. If S is the area of a triangle ABC, prove that

$$S = \frac{1}{4}(a^2 \sin(2B) + b^2 \sin(2A)),$$

where a and b are the lengths of the sides opposite the angles A and B, respectively.

Q1189. Prove that if A, B and C are three angles of a triangle then

$$\frac{\sin A + \sin B + \sin C}{\cos A + \cos B + \cos C} < 2.$$

Q1190. Prove that for any positive integer n,

$$\frac{1}{2\sqrt{1}} + \frac{1}{3\sqrt{2}} + \frac{1}{4\sqrt{3}} + \cdots + \frac{1}{(n+1)\sqrt{n}} < 2.$$