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PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q1064 The numbers $1, 2, \ldots, 16$ are placed in the cells of a 4×4 table as shown in the left hand diagram below. One may add 1 to all numbers of any row or subtract 1 from all numbers of any column. How can one obtain the table as shown in the right hand diagram below using these operations?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16

Q1065 In the land of OZ the national currency has notes of value $1, $10, $100 and $1000. Is it possible to own exactly half a million notes with a total value of $1 million?

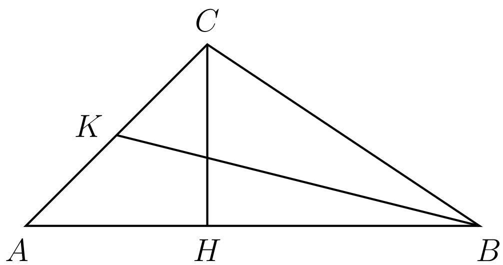
Q1066 The king of OZ intends to build six fortresses in the country and to connect each pair of fortresses by a two-way road. Show that it can be done so that there would be exactly three intersections and exactly two roads would cross at each intersection.

Q1067 If each boy purchases a Mars bar and each girl purchases a cupcake, they would spend a total of one cent more than if each boy purchased a cupcake and each girl purchased a Mars bar. We know that there are more boys than girls. How many more boys than girls are there?

Q1068 Melbourne tram tickets have six-digit numbers (from 000000 to 999999). A ticket is called lucky if the sum of its first three digits is equal to the sum of its last three digits. How many consecutively-numbered tickets should one buy to be sure of getting at least one lucky ticket, assuming that one does not know where the sequence will start.

Q1069 Two players play the following game on a 9×9 chessboard. They play alternatively and the first player places a black counter in a square and then the second player places a white counter in a square. The game finishes when all 81 squares are filled with counters. There are 9 rows and 9 columns and so there are either more white counters or more black counters in each row and column. Each player gets a point if they have more counters in a row or a column. So the total number of points allocated is 18. What is the highest number of points the first player can gain if both players play as well as possible?

Q1070 The altitude CH and the median BK are drawn in the acute angled triangle ABC, and it is known that $BK = CH$ and $\angle KBC = \angle HBK$. Prove that the triangle ABC is equilateral.



Q1071 The national currencies of Dillia and Dallia are called the diller and the daller, respectively. The exchange rate is such that in Dillia one diller can be exchanged for ten dallers, and in Dallia one daller can be exchanged for ten dillers. A young girl has one diller and can change her money in either country free of charge. Prove that she will never have equal numbers of dillers and dallers.