Parabola Volume 55, Issue 2 (2019)

Problems 1591–1600

Parabola thanks Shiva Oswal for contributing Problems 1594, 1595 and 1596.

Q1591 For

$$f(x) = x^4 + 2x^3 - 7x^2 + 11 ,$$

find a line which is tangent to the graph $y = f(x)$ twice.

Q1592 You have an unfair coin for which heads turns up with probability $p > \frac{1}{2}$. You flip the coin repeatedly until there have been more heads than tails. How many flips, on average, does this take?

Q1593 A 3×5 chessboard has three red counters in the leftmost column and three blue counters in the rightmost column. A counter can move to an adjacent square vertically or horizontally. Moves alternate between the two colours, and no two counters may occupy the same square simultaneously. Move the red counters into the right column and the blue counters into the left column in the minimum possible number of moves.

Q1594 Briana arranges square unit tiles in a special way. She begins with a single tile, then puts one tile on its right to form a rectangle; and then a row of tiles from left to right along the top to form a square. She then puts two tiles on the right (from bottom to top) to form a rectangle, and another row on top to form a square; and so on. For example, the diagram indicates the order of placement of the first 11 tiles.

7	8	9	
3	4	6	11
1	2	5	10

What is the perimeter length of Briana's figure after the first 2019 tiles have been placed?

Q1595 A regular tetrahedron has a total surface area of $16\sqrt{3}$. Find the sum of the lengths of the edges of this tetrahedron.

Q1596 Robert has n marbles in a jar, each either blue or red. At the start there are equal numbers of blue and red marbles in the jar. Robert draws a marble from the jar twice without replacement. If the probability that both the marbles drawn are red is $\frac{2}{9}$, find n.

Q1597 Find the highest common factor of the integers

$$m = 2^{20} + 3^{19} \quad \text{and} \quad n = 2^{19} + 3^{20} .$$

Q1598 If the cubic polynomial

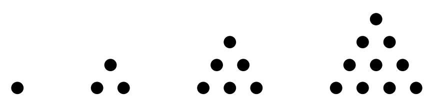
$$f(x) = x^3 + 2x^2 + 3x + 4$$

has roots a, b, c, then find a cubic polynomial with roots ab, bc, ca.

Q1599 The **triangular numbers** are

$$T_1 = 1,\ \ T_2 = 1 + 2 = 3,\ \ T_3 = 1 + 2 + 3 = 6,\ \ T_4 = 1 + 2 + 3 + 4 = 10$$

and so on; we also define $T_0 = 0$. They can be illustrated as the numbers of dots which can be formed into equilateral triangular shapes:



Let n be a positive integer. Prove that the number of ways of writing n as a difference of two triangular numbers, $n = T_a - T_b$, is equal to the number of odd factors of n.

Q1600 A social network has $2n$ members. Any two members are either friends or not. There are no three members p, q, r that are all friends with each other.
Prove that there are at most n^2 unordered friend–pairs.
(Here, $\{p, q\}$ and $\{q, p\}$ count as the same friend-pair.)