YOUR LETTERS

Dear Sir,

Recently I've bought a tiny calculator and discovered the following ways of doing rapid calculations:

(1) To square a number ending in 5, write down 25 and then multiply the digit in the tens column by the next digit.

E.g. For 55^2, $5 \times (5 + 1) = 30$ and so $55^2 = 3025$.

Multiplying two numbers with the same digit in the tens column where the digits in the units column add up to 10, is done virtually the same way.

E.g. To find 46×44, write down $6 \times 4 = 24$.

In tens column $4 \times (4 + 1) = 20$ and so $46 \times 44 = 2024$.

Experimenting with my calculator, I've found a couple of other interesting things.

(2) To multiply two numbers where the digits in the units column add up to 10 and the digits in the tens column differ by 1, and the difference between the numbers is eight or less, use the following method. Multiply the digits in the units column and write down the digit in the units column of the answer, write down 9 (unless the units columns were 6×4, when you write down 8), and next multiply the smaller digit in the tens column by one more than the larger digit.

E.g. For 42×38

$8 \times 2 = 16 \to 6$

$3 \times (4 + 1) = 15$

Answer = 1596

and 46×54,

$6 \times 4 = 24 \to 4$

$4 \times (5 + 1) = 24$

Answer = 2484

(3) Another example is when the digits in the tens columns are the same and the digits in the units column add to 5. Multiply the digits in the units column and write down the answer. If the digit N in the tens column is even, write down 0 and write $N^2 + \frac{1}{2}N$ in front of it; if N is odd, write down 5 and write $N^2 + \frac{1}{2}N - \frac{1}{2}$ in front of it.

E.g. For 41×44

$1 \times 4 = 4$

$4 + \frac{1}{2} \times 4 = 18$

Answer = 1804

and 52×53

$2 \times 3 = 6$

$5 + \frac{1}{2} \times 5 - \frac{1}{2} = 27$

Answer = 2756.

Yours sincerely,

Jimmy Pik[e]

Dear Sir,

A very useful way to check the result of a multiplication is probably unknown to many, though it is quite simple. It was explained to me by my uncle.

After you have completed the main multiplication, take the first line of digits and add them together. Divide this by nine and write down the remainder. Then take the second line of digits and do the same. Multiply these two remainders together to obtain answer A. If the digits of the answer to the main multiplication are then added, divided by nine and the remainder noted, you will find, if your result to the main multiplication is correct, that the remainder is the same as answer A. If it is not, you have made a mistake somewhere.

Example 1.

5762 x	5 + 7 + 6 + 2	= 20	$20 \div 9$ has remainder 2 x
45	4 + 5	9	$9 \div 9$ has remainder 0
259290	2 + 5 + 9 + 2 + 9 + 0 = 27		$27 \div 9$ has remainder 0 = Answer A

Example 2.

863 x	8 + 6 + 3	= 17	$17 \div 9$ has remainder 8 x
44	4 + 4	= 8	$8 \div 9$ has remainder 8
37972	3 + 7 + 9 + 7 + 2	= 28	$28 \div 9$ has remainder 1
For check:		8 x 8 = 64	$64 \div 9$ has remainder 1.

For cases where the two remainders' product is greater than nine, divide by nine and then note the remainder after this. (See example 2.)

Philip Zammit,
Form II,
St. Joseph's College

[This method is called "casting out the nines." It is done by noting that the remainder after dividing by 9 can be found by adding the digits of the number. E.g. 64 has remainder 6 + 4 = 10 which has remainder 1 + 0 = 1 — Editor.]

Dear Sir,

The following is a formula which I have derived for the area of a triangle ABC with sides of length a, b, c.

Let $\quad x = b^2 + c^2 - a^2$ ... (1)

and $\quad y = 2bc$ ... (2)

Then, by the cosine rule, $\cos A = \frac{x}{y}$

$$\sin A = \sqrt{1 - \cos^2 A} \;=\; \sqrt{y^2 - x^2}/y.$$

So area of triangle $= \frac{1}{2}\, bc \sin A$

$= bc\sqrt{y^2 - x^2}/2y$

$= \sqrt{y^2 - x^2}/4$ by (2)

$= \sqrt{(2bc)^2 - (b^2 + c^2 - a^2)^2}/4$ by (1) and (2).

Garry Helprin,
Cranbrook.

As well as the above letters, I have received letters from eight students at Grafton High School. They all say that Parabola, although interesting, is too hard for all except the first level senior students. As editor, I would appreciate your views on this together with suggestions as to *what you want.* Here are some excerpts from some of the letters from Grafton High.

"If the magazine were broadened and included articles of interest (e.g. mini-calculators) . . . it would be a better magazine and of more value to me."

[We will see what we can do about mini-calculators. Any other suggestions? — Ed.]

"In my opinion there should be more games and more puzzles to make the magazine more interesting. . . .
Thirdly, I was disappointed when I first opened up the magazine and saw page after page of black print on white paper. It would be wiser to brighten the pages up a bit (Perhaps you could use some illustrations?)."

Jenny Stanton

I agree with Jenny, but we have very little artistic talent on our staff. However, all contributions will be gratefully received and printed. — Ed.]

"My suggestion is that the printing of solutions [to the problems] be delayed by one issue, and in the intervening issue a special 'Hints' section be introduced."

This raises the problem that the solution to problems in Nos. 2 and 3 of any year would not appear until the next year. I would like to hear the views of other readers. — Ed.]

"I would prefer it to be published in volumes for each form – coinciding with the maths the student will approach during the year. The magazine should aid a textbook which the student has, explain in much more depth the problems and how to find the solutions, and discuss the various mathematical quests the student will confront in each form respectively.

'Your Letters' should be kept so students can write in and be helped with problems."

Warren Kennedy

[Our aim is not to help you with your schoolwork but give you some "extra-curricular" Maths. However, I will greatly welcome any problems with which you want any help. — Ed.]

We cannot give you what you want unless you tell us what it is.

A "formula" for the n'th prime

In Vol. 10, No. 1, it was stated that there was no known formula for the n'th prime. This is not strictly correct. There *is* a formula in the sense that it enables one to calculate the n'th prime in a finite number of steps from the preceding primes. (No-one would use it, of course!)

Let Q be the product of the first n–1 primes ($n \geqslant 2$). Then the n'th prime is the unique integer p such that

$$1 < 2^p\left(-\tfrac{1}{2} + \sum_{d|Q} \frac{\mu(d)}{2^d - 1}\right) < 2$$

where the Σ sign means to add the expression for all the factors d of Q and

$\mu(1) = 1$

$\mu(d) = (-1)^r$ where r is the number of primes which have been multiplied together to get d ($d \neq 1$).

E.g.

$$\mu(2) = \mu(3) = \mu(5) = \mu(7) = -1$$
$$\mu(6) = \mu(2 \times 3) = (-1)^2 = 1.$$

For a proof of this result, see the American Mathematical Monthly, Vol. 79, page 625.

J. Mack