PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year or form. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

Q.913 S is a collection of numbers. At least half of the numbers in S are even, at least two thirds are multiples of 3, and at least six sevenths are multiples of 7. Prove that at least one of the numbers in S is a multiple of 42.

Q.914 In the following addition, the letters $A, B, \ldots, I$ represent single digits, all different.

$$\begin{array}{rccc} & A & B & C \\ + & D & E & F \\ + & G & H & I \\ \hline 1 & 9 & 9 & 4 \end{array}$$

In how many ways can the sum be reconstructed?

Q.915 A triangle has sides of length 13, 30 and 37. Find all possible triangles with sides 13 and 30, a different third side, and the same area as the given triangle.

Q.916 Find two ten-digit numbers such that the first digit of each number is the number of ones in the other, the second digit in each is the number of twos in the other, and so on, the tenth digit in each being the number of zeros in the other.

Q.917 We play a game as follows: a coin is tossed repeatedly; for each head you score one point and for each tail I score one point. The first to seven points wins \$10 from the other. When the score reaches 6-4 in your favour, I note that you have won 6 points out of 10, and offer you \$6 to abandon the game. Should you accept? What if the winner were to be the first player to reach ten points, all other conditions remaining the same?

Q.918 The digits $1, 2, 3, 4$ are to be arranged to form a number, using *no* extra symbols. Two methods are permitted: joining digits to form a multi-digit number, and using power notation. For example some possible constructions are

$$123^4, \ 41^{32} \quad \text{and} \quad 3^{21^4}.$$

(Note that the last means $3^{(21^4)}$, not $(3^{21})^4$: such an expression is evaluated from the top down.) What is the largest number that can be formed under these conditions?

Q.919 The Australian No-Hopers Party (ANHP) has 25 members in Parliament, including exactly two who live in Canberra, ride bicycles and own whiteboards. It is known that

(a) if the number of ANHP members who live in Canberra and own whiteboards is greater than 4 or the number who ride bicycles but do not live in Canberra is greater than 3, then the total number who own whiteboards is 15;

(b) if the number who ride bicycles is less than 9 or the number who do not own whiteboards is greater than 7, then the number who both ride bicycles and own whiteboards is 8;

(c) if the number who live in Canberra but do not ride bicycles is not equal to 12 or the number who ride bicycles and own whiteboards is less than 8, then the total number who ride bicycles is 4.

If you knew the total number of ANHP members who ride bicycles, you could calculate how many live in Canberra. Determine how many members of the ANHP own whiteboards, how many live in Canberra and how many ride bicycles.

Q.920 The well-known Fibonacci sequence consists of the numbers $F_1, F_2, F_3, \ldots$ where $F_1 = F_2 = 1$ and each subsequent number in the list is the sum of the previous two: that is, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Show that one of the first million Fibonacci numbers is a multiple of 997.

Q.921 A supermarket runs a free contest for its customers, who have to collect numbered tickets adding up to 1994 or 1995. For 1994 the customer wins \$100; for 1995, \$1000. The company can print any number of tickets, with any integers on them, but in order to create public interest they wish to print a large variety of different tickets. However they also wish to be *certain* that at most ten \$1000 prizes and fifty \$100 prizes can be won. Devise a scheme to achieve this.

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