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AFTER FERMAT — WHAT NEXT?

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School textbooks often give the impression that Mathematics has all been worked out. In fact, nothing could be further from the truth. Each year, there are many thousands of publications containing new mathematical results. Some of these are of course very esoteric. Nonetheless, there are still some easily stated problems that no-one knows how to do. Until recently, the most famous of these was probably *Fermat's Last Theorem*. This states that not only can't you find any positive integers x, y and z such that $x^3 + y^3 = z^3$, but you can't do this for any higher powers either. That is, if $n \geq 3$ then the equation

$$x^n + y^n = z^n$$

has no positive integer solutions. Mathematicians had been trying to prove this for about 400 years until Andrew Wiles from Princeton University recently came up with a proof. When Wiles announced his proof on 23 June 1993, the news was flashed to mathematicians around the world. Actually, a flaw was found in his original proof, but this has since been fixed, and the 200 page proof has just appeared in print.

What problems are left then for a mathematician aspiring to fame (if not fortune)? Not quite as old as Fermat's Last Theorem is the *Goldbach Conjecture*. This states that every even integer greater than or equal to 4 can be written as the sum of two prime numbers. That is, if $n \geq 4$ is even, then $n = p_1 + p_2$ where p_1 and p_2 are prime. It is easy to check the first few:

$$4 = 2 + 2, \qquad 6 = 3 + 3, \qquad 8 = 3 + 5, \qquad 10 = 5 + 5, \qquad 12 = 5 + 7, \qquad etc.$$

For bigger even numbers it is not always so easy: try writing 992 as the sum of two prime numbers[2]. The conjecture that you can always split up an even number in this way is due to Goldbach, who having checked the first few hundred even numbers, asked the great Swiss mathematician Leonhard Euler in 1742 if he could prove that this was always the case. Euler never solved the problem, and neither has anyone since.

You can of course always write an even number as a sum of prime numbers if you take enough of them: $1000000 = 2 + 2 + \cdots + 2$. In this case we could have been more economical since $1000000 = 17 + 999983$ and 17 and 999983 are both prime. For a long time however, no-one could **prove** that you didn't need to take more and more prime numbers as n gets bigger. That fact at least was proved by a Russian mathematician Schnirelmann in 1931. The best result so far is due to another Russian, Vinogradov:

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[2] $992 = 97 + 919$ works — and you can't use a smaller number than 97.

Theorem (Vinogradov's Theorem)**.** *There is a integer N such that every integer (odd or even) larger than N is the sum of at most 4 prime numbers.*

The problem is that no-one knows how big N is! Of course if Goldbach's Conjecture is true then $N = 1$.

Good Luck!