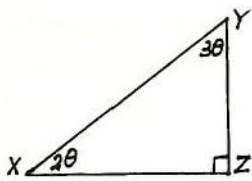


## The value of $\sin \frac{\pi}{10}$

Peter Kanka, of James Cook Boys Technology High School, provided the following proof that  $\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$  when solving Question 945.



In  $XYZ$ ,  $\angle XYZ = 90^\circ$ ,  
 $XY = 1$  unit,  $\angle YXZ = 2\theta$  and  $\angle XYZ = 3\theta$  (hence  $5\theta = 90^\circ$   
 and  $\theta = 18^\circ$ ).

Now,  $XZ = \cos 2\theta = \sin 3\theta$  ..... (i)

and  $YZ = \sin 2\theta = \cos 3\theta$  ..... (ii)

We can now find  $\sin \theta$  from either equation (i) or (ii). Let us choose equation (ii),

$$\therefore 2 \sin \theta \cos \theta = 4 \cos^3 \theta - 3 \cos \theta$$

so that  $\cos \theta (4 \cos^2 \theta - 2 \sin \theta - 3) = 0$

$$\therefore 4 \sin^2 \theta + 2 \sin \theta - 1 = 0 \quad \text{since } \cos \theta \neq 0.$$

Using the “quadratic formula”, and noting that  $\sin \theta > 0$ , we find that the exact value of  $\sin 18^\circ$  is  $\frac{\sqrt{5} - 1}{4}$ .