PROBLEM SECTION

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions of these problems will appear in the next issue of **Parabola**; your solution(s) may be used if they are received in time.

You are also invited to send in possible problems for inclusion. For example, problems Q1100, Q1101 and Q1103 were sent in by Peter Merrotsy of Kyogle.

Q1098 Consider the 2000-digit number which begins with 1000 4's, next has 999 8's and ends in a 9. Find its square root.

Q1099 I have three discs, each with one number on each side. Playing 3-up the totals 12, 13, 14, 15, 16, 17, 18 and 19 occur equally often. When the total was 18, the numbers on the three visible faces were 5, 6 and 7. Can you work out what numbers were written on the reverse sides? Is there just one solution, or are there several solutions? Is it possible, by starting with three suitably numberered discs, to obtain *any* eight consecutive numbers as totals in this way?

Q1100 (a) $ABCD$ is a square. The point X is constructed so that

$$\angle BAX = \angle ABX = 15°.$$

Show that $\triangle CDX$ is equilateral.
(b)
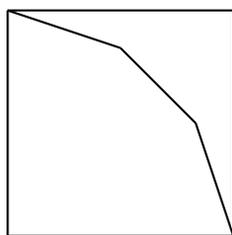
A square is constructed on a regular dodecagon (twelve sided polygon) so that one vertex is at the centre of the dodecagon and two vertices of the square coincide with two vertices of the dodecagon, as in the diagram.

Using part (a) of this problem, calculate the ratio of the area of the square to the area of the part of the dodecagon it covers.

Q1101
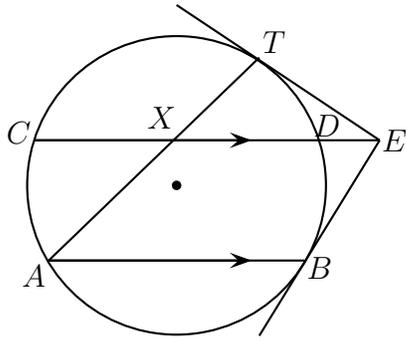
Two chords AB and CD in a circle are parallel. The tangent at B meets CD produced at E. The other tangent which passes through E meets the circle at T. AT meets CD at X. Show that $CX = XD$.

Q1102 Find the first positive integer whose square ends in three 4s. Find all positive integers whose square ends in three 4s. Show that no perfect square ends in four 4s.

Q1103 In households all over Australia squares of cloth are often folded as follows:
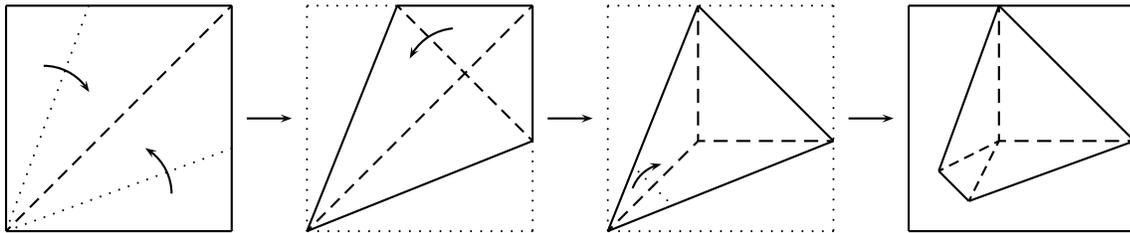
What is the ratio of the area of the folded area to the area of the original square?

Q1104 $ABCDEFGH$ is a cube of side 2.

(a) Find the area of the quadrilateral $AMHN$, where M is the midpoint of BC, and N is the midpoint of EF.

(b) Let P be the midpoint of AB, and Q the midpoint of HE. Let AM meet CP at X, and let HN meet FQ at Y. Find the length of XY.

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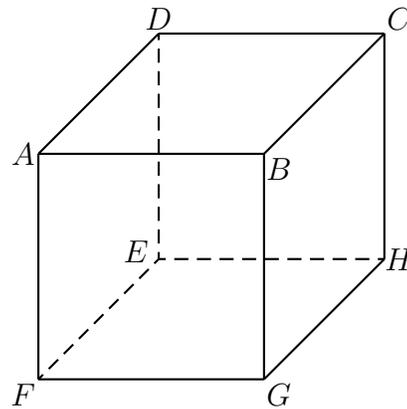
A number of the problems used in this year's problem sections are from two excellent books by Tony Gardiner:

The Mathematical Olympiad Handbook, An Introduction to Problem Solving published by Oxford University Press in 1997 and

Mathematical Puzzling published by the United Kingdom Mathematics Foundation in 1996.