Parabola Volume 39, Issue 1 (2003)

Problems Section

You are invited to submit solutions to any or all of the following problems, accompanied by your name, school and year. Solutions to these problems will appear in the next issue of *Parabola*; your solution(s) may be used if they are received in time.

Q1131 Suppose A and B are two equally strong tennis players. Is it more probable that A will beat B in three sets out of 4 or in 5 sets out of eight?

Q1132 Triball is a three player game. In each round, the winner scores a points, the runner up is awarded b points, and the loser gets c points, where $a > b > c$ are positive integers. One day Xavier, Yvonne and Zachary played some triball and the final score was

$$\text{Xavier} - 20 \qquad \text{Yvonne} - 10 \qquad \text{Zachary} - 9.$$

Yvonne won the second round. Who won the first round, and how many points did Zachary get in the last round?

Q1133 Let us assume that in a knockout tennis tournament the players always play true to form, so that the stronger player invariably defeats a weaker opponent. If there are 16 entrants in the tournament, and the draw is decided by lot, what is the probability that the final (in round 5) is played between the strongest two players?

Q1134 Let $a_1, a_2, \ldots, a_n$ be positive numbers. Prove that

$$\frac{a_2 + a_3 + \cdots + a_n}{a_1} + \frac{a_1 + a_3 + \cdots + a_n}{a_2} + \cdots$$
$$\cdots + \frac{a_1 + a_2 + \cdots + a_{n-1}}{a_n} \geq n^2 - n\ ,$$

and deduce that

$$\big(a_1 + a_2 + \cdots + a_n\big)\Big(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\Big) \geq n^2\ .$$

Q1135 Prove the rearrangement inequalities for products of sums.

Q1136 Arrange the numbers $1, 2, \ldots, 2n$ into an order $x_1, x_2, \ldots, x_{2n}$ so as to obtain the maximum value of

$$x_1x_2 + x_3x_4 + x_5x_6 + \cdots + x_{2n-1}x_{2n} \,.$$

Also, find the minimum value of the sum.

Q1137 At a party, there are $2n$ people, of whom each one is acquainted with at least n others. Prove that it is possible to seat them at a circular table so that each is seated between two acquaintances.

Q1138 Two bodies move in opposite directions around a circular track, one at constant speed v m/sec. The speed of the other increases at a constant rate a m/sec/sec. At time $t = 0$, the bodies are of the same point A and the second one is momentarily at rest. In how many seconds does their first meeting occur, if their second meeting is again at the point A?

Q1139 A mixed school organizes a chess competition in which every participant plays one game against every one else. At the conclusion of the tournament, it is noted that each competitor gained exactly half of his or her points in games against boys. (As usual, a win earns 1 point, and a draw earns $\frac{1}{2}$ point for both players.) Prove that the total number of competitors is a perfect square.

Q1140 Fewer than 135 players entered a tournament in which each match has three contestants and one winner. Before each round the names of all the survivors were drawn to provide as many matches as possible, with byes being given to any players left out. It turned out that the tournament was won outright by a player who contested only one match. What was the greatest possible number who took part?