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Finding Multiple Solutions to Tangrams

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Many of us have been delighted with the extraordinary geometrical forms that can be constructed with the seven pieces of the rectangular or the square tangrams. The rectangular tangram, which is considered in this article, consists of four quadrilaterals (trapezoids), two rectangular triangles (isosceles), and one pentagon. This tangram is shown in Figure 1 below.

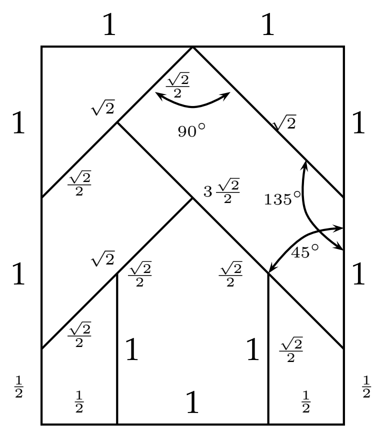

Figure 1

Note that the corner angles of all the pieces (called 'tans') are multiples of 45 degrees.

The classic rules to play tangrams are as follows: All seven 'tans' must be used to form a given shape or figure. The tans must touch, but they must not overlap. The tans must lay flat.

Trying to find as many possible solutions to a given shape is one of the most challenging ways to play tangrams. This article shows how rotational symmetry, virtual images and canonical shapes can be used to proliferate further solutions from known solutions.

Rotational Symmetry

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Many tangrams have a symmetrical shape. Also, when a solution to a given tangram is obtained, it can be observed that in certain cases, inside that solution you could see a symmetrical shape formed by two, three and more 'tans' similar to the ones shown below:

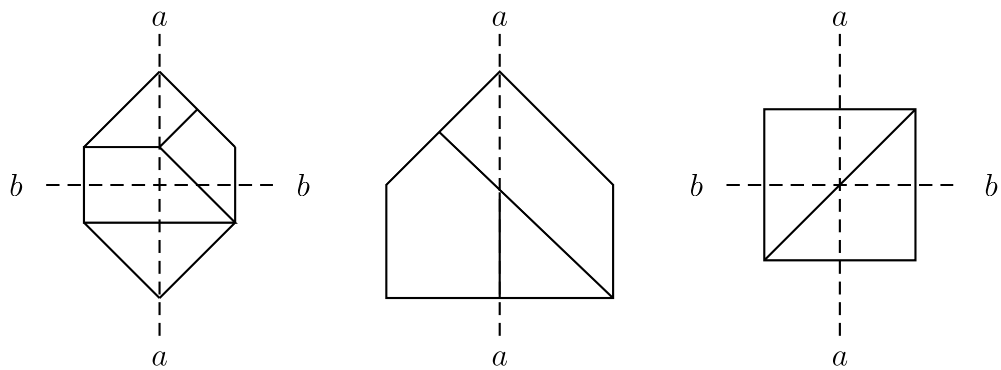

Figure 2

Each of the above shapes remains invariant after rotation through $180°$ about the $a - a$ or $b - b$ axes indicated.

Figure 3 shows a shape (left) that can be composed of tans in different ways (A,B,C right).

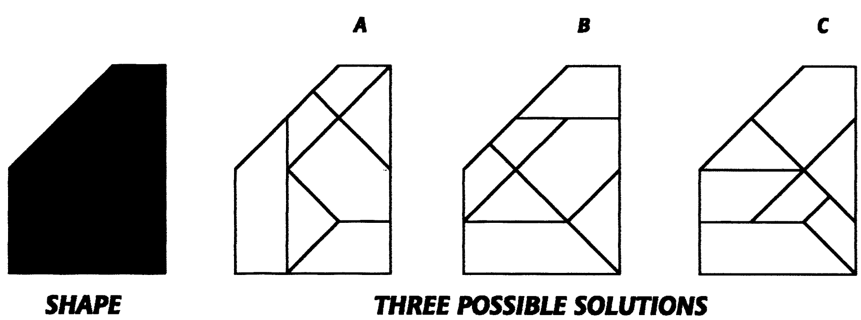

Figure 3

Note that inside the different solutions there are certain rotationally symmetrical shapes. For example, inside solutions (A) and (C), we can see the five rotationally symmetrical shapes and their symmetry axes shown below:

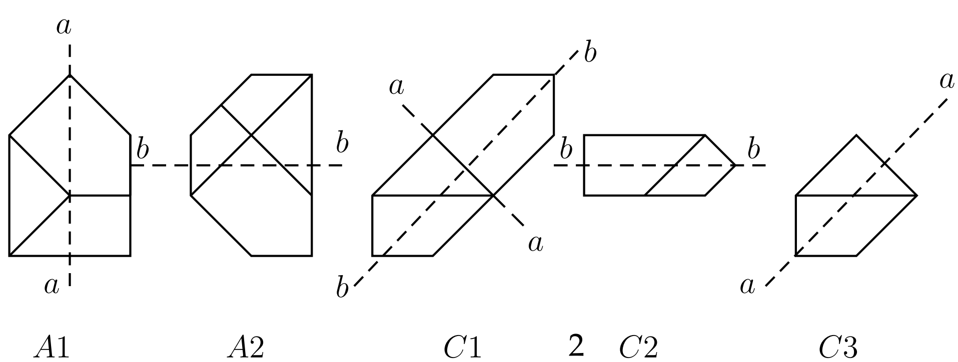

Figure 4

With figure 3B, we can see that it too contains another rotationally symmetrical shape. In this case the symmetry axis is orthogonal to the $a - a$ and $b - b$ axis shown.

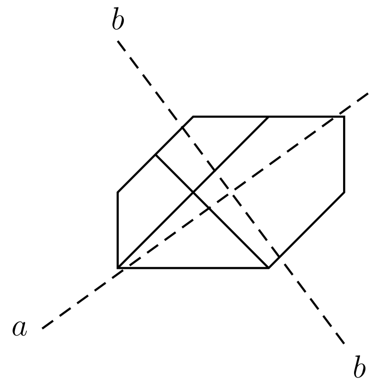

Figure 5

Thus once the shape shown in Figure 5 is detected we can immediately obtain three further solutions.

Figures 6 and 7 show further examples of rotational symmetry. In these figures the x and y axes go along the width and length of the page respectively and the z-axis is perpendicular to the paper and points to the reader.

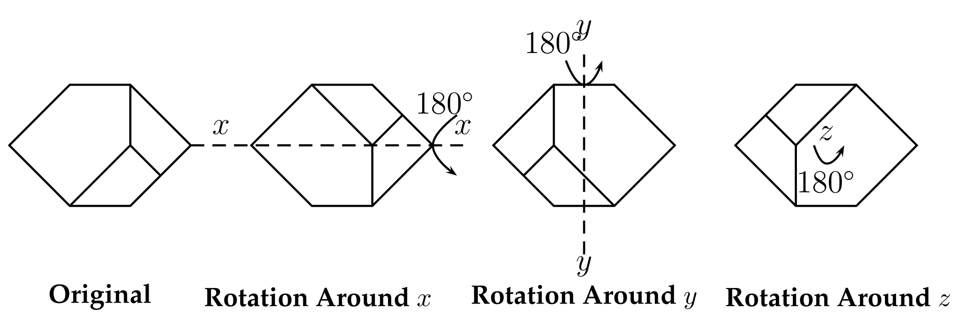

Figure 6

Note that in Figure 7, only one different solution with the three rotations is obtained, because the rotation around the z-axis gives us the same solution as before and the rotation around the axes x and y are both exactly the same.

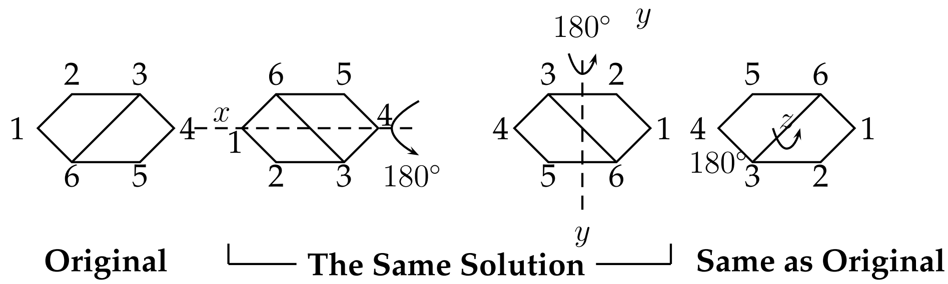

Figure 7

Units that Generate Solutions

The rotationally symmetrical shapes that can be seen inside a particular solution could be used in assisting us to find many of the remaining possible solutions to a given tangram. These shapes are obtained when we have two, three, four or more 'tans' arranged as shown:

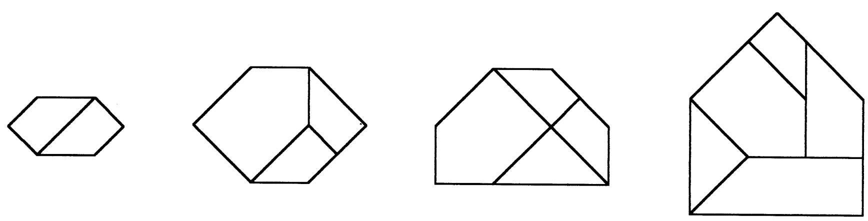

Figure 8

Due to the fact that these shapes can be used to create new and different solutions, we name them "Solution Generator Units", or SGU for short. When we detect any of these shapes, we will be able to use them to generate new and different solutions. If they have two different axes of symmetry and depending on the form itself, it will be possible to have with their assistance, two or even three more solutions.

The next figure shows more than 60 SGUs that could be detected and which can be used to find new solutions. All of them have been assigned the letter "G" for generator.

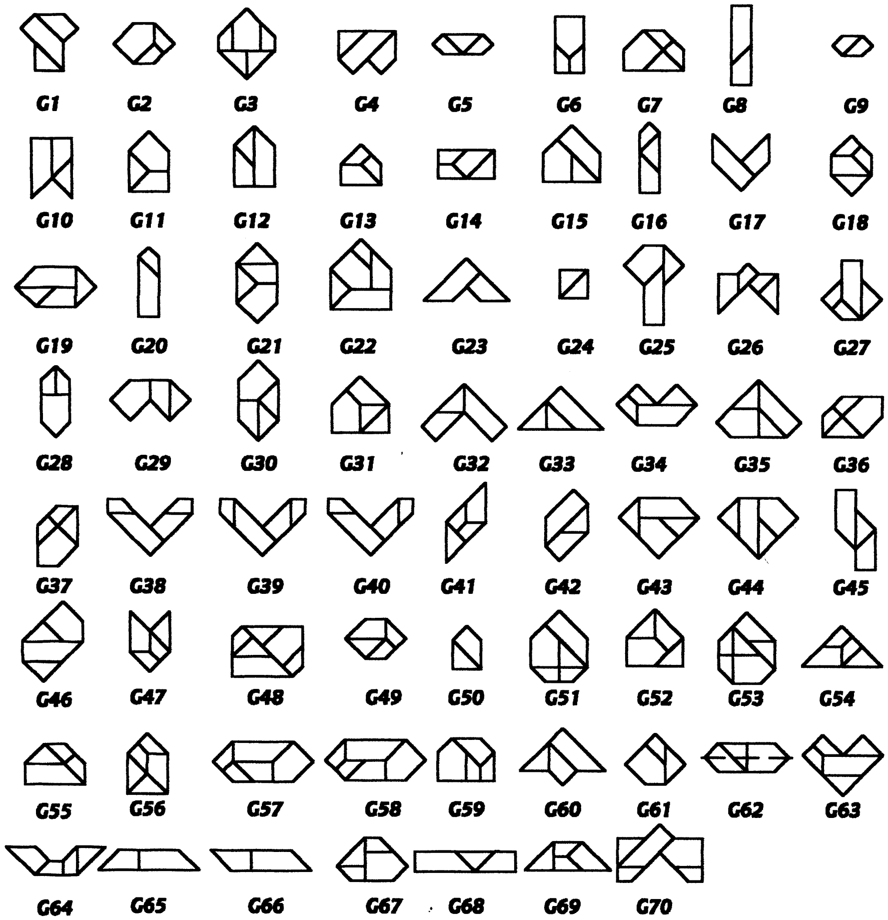

Figure 9

Real and Virtual Images of Tans and Groups of Tans

The shape of some 'tans', like the pentagon or the large trapezium of the rectangular tangram could be reproduced using some other 'tans' of the same set. We have given the pentagon and the large trapezium the name of 'real images'. To the reproduced shapes, formed by using the other 'tans', we have given the name of 'virtual images'. The use of this technique of finding virtual images is also useful for finding more solutions to a given tangram. For example, by using the medium size trapezium and one of the triangles we can reproduce the pentagon. Also, using the medium size trapezium and the other triangle, as shown below, the large trapezium could be reproduced.

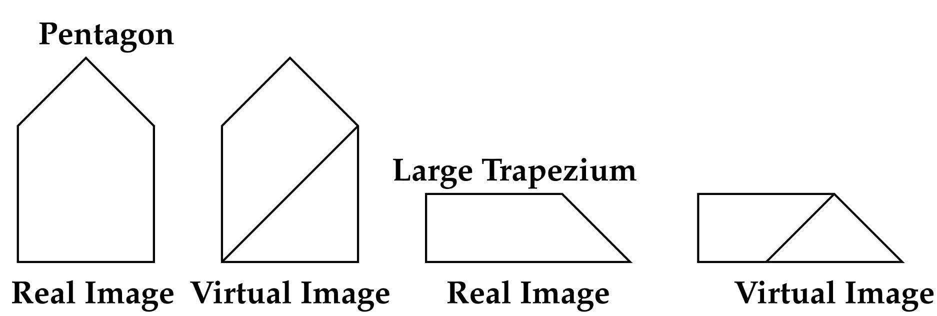

Figure 10

The names of real and virtual images could also be used with other groups of 'tans' that are obtained by joining two, three or more 'tans'. Their 'virtual image' can be seen drawn next to the 'real image' as both are identified below.

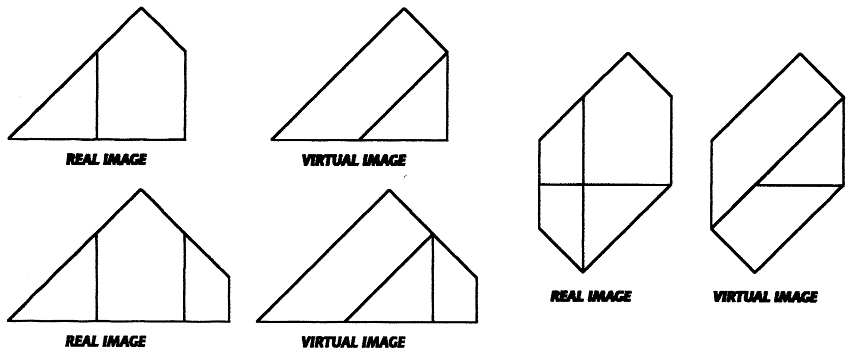

Figure 11

Asymmetric Units

Sometimes it is possible to find inside a solution to a tangram, certain shapes similar to the ones shown below, made up of three or more 'tans'.

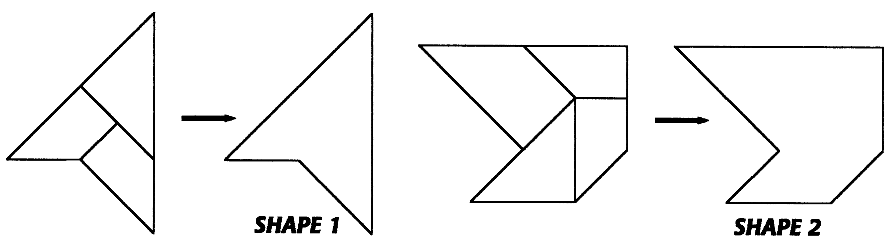

Figure 12

The same figures, shape 1 and shape 2 could be obtained using the same group of 'tans' in each case, but positioning them differently inside the shapes as shown below:

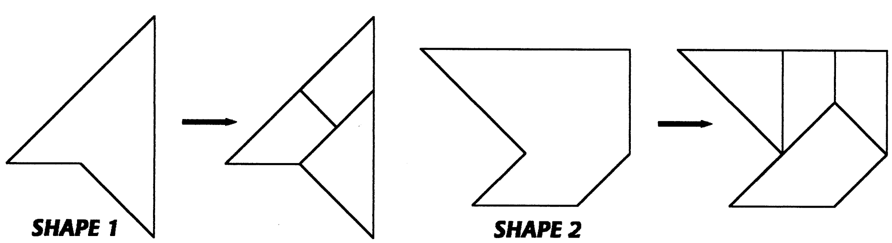

Figure 13

For this reason, these shapes when found, will help us to find new solutions. They are not as common as the SGU and they are not symmetric, and it is for this particular reason that I had called them asymmetric units.

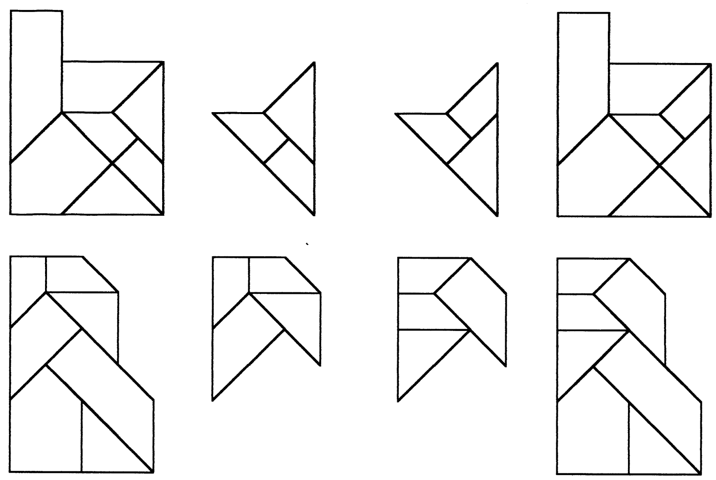

Figure 14

The figures that conclude this article show twenty three different possible arrangements of the tans to produce the basic house shape.

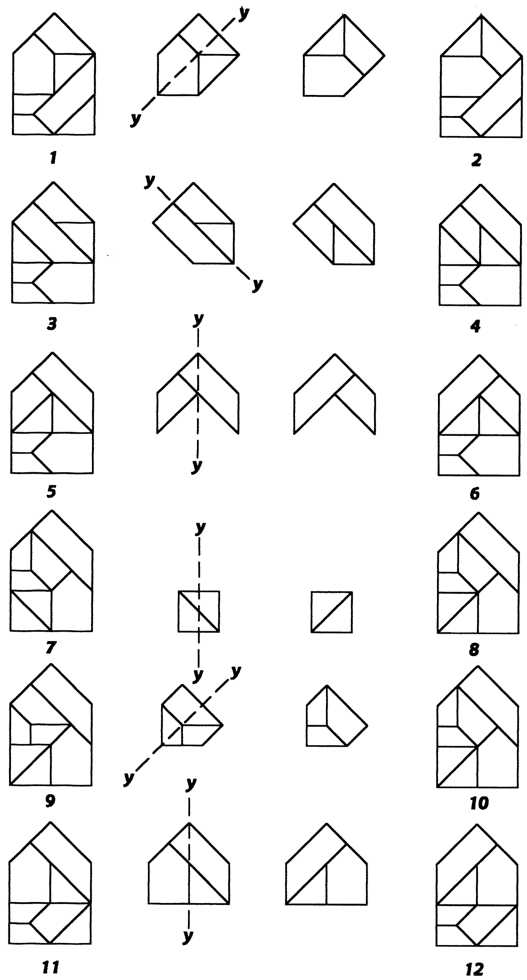

Figure 15a

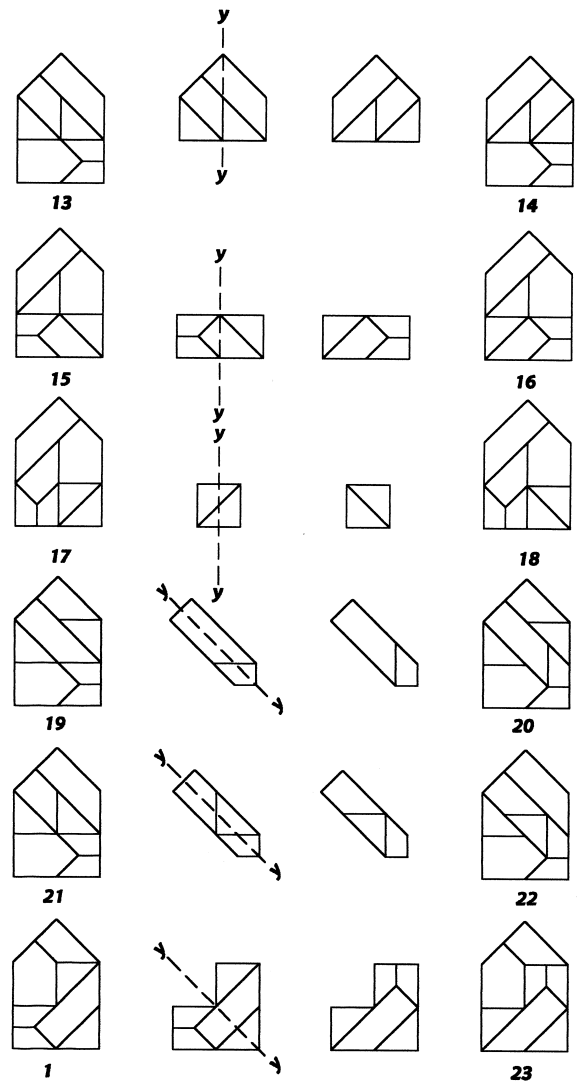

Figure 15b