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Review of Mathematica 5.0.0

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My first experience with an algebraic manipulation package was about twenty years ago, toward the end of my PhD in theoretical physics. I remember that first time I used `REDUCE` to differentiate and simplify a very long expression that had taken me days to do by hand. Within a few minutes it was done and I wanted to rush out into the corridors and tell everybody about it. It was an overwhelming feeling of having new mathematical power. At this time in my career I was running large numerical codes in `FORTRAN77` on a large main frame PDP11 computer to simulate motions of atoms subject to various inter-atomic forces. I was using `UNIRAS` for plotting graphs and I was using an Apple II computer for (text only) word processing. I can recall dreaming of the day when all of these things could come together. That day dawned a little over ten years ago when MATHEMATICA 2 was released combining algebraic manipulations, numerical solvers, graphics and word processing (notebooks) in a single package. But of course there were tandem developments in improved specialist packages for solving problems numerically and others for manipulating symbols algebraically and others for graphics and word processing and so on. Like all mathematical researchers I wanted the best in everything and so MATHEMATICA 2 as brilliant as it was didn't replace the suite of packages that I was using, although it did take care of all my algebraic symbol manipulation and plotting needs.

MATHEMATICA 5, on the other hand, is close to making all other packages obsolete. A PC loaded with MATHEMATICA 5 is far superior to the combined assortment of main frames, (including supercomputers), numerical routines, numerical programming languages, symbol manipulation packages, graphics packages and word processing packages that I had to call on during my PhD and PostDoc years fifteen to twenty years ago. But even more MATHEMATICA 5 is comparable with or better than any of its present day competitors in algebraic symbol manipulation, graphics and numerical computations.

In its early days MATHEMATICA was used mainly by specialist researchers in the physical and mathematical sciences for its algebraic symbol manipulation capabilities. MATHEMATICA has since evolved to meet the needs of a broader technically minded user group. This broader group includes financial mathematicians, mathematical biologists and data miners. In addressing the needs of this broader group, MATHEMATICA has put even more emphasis into developing high powered functions with in built defaults that in turn call on many different algorithms depending on the nature of the problem. In this way the user needs to know the area of mathematics that they need

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technical assistance with but they do not need to know the detail of the assistance being provided. For example an epidemiologist modelling the spatial and temporal spread of an infectious disease might set the problem up in terms of partial differential equations subject to certain boundary conditions and then use the command `NDSolve` on MATHEMATICA. This technically minded user does not need to know the detail of which of the more than thirty algorithms MATHEMATICA might select to solve the problem. In MATHEMATICA 5 there is also a far greater emphasis on numerical problem solving than there has been in past versions. Again this is motivated by the recognition that the broader user group is more interested in addressing real world problems than it is in investigating model problems. In real world problems numerical simulations over many different model parameters are generally required.

MATHEMATICA 5 is an easy product to use. The syntax that MATHEMATICA uses for scientific functions has a natural feel for technically minded users (e.g., `FullSimplify` means fully simplify and `NMinimize` means numerically minimize and `FindFit` is a superfunction to find the best fit algebraic expression for a set of data points). It is also easy to enter complicated expressions either directly or with assistance from the palettes provided. The inbuilt help in MATHEMATICA is also vast and comprehensive and particularly useful in picking up syntax errors or common mistakes, although certain naive errors such as `Int[Sin[x],x]` instead of `Integrate[Sin[x],x]` may go undetected as an error – suggesting a command syntax error if it did not yield the expected type of output. This is where the command completion utility of MATHEMATICA can prove very useful. For example if you typed `Int` followed by CONTROL+K you would obtain a list of all MATHEMATICA commands starting with `Int` and it would be obvious from the list that `Integrate` is what you should have used.

Another really neat feature of MATHEMATICA is that the output from a MATHEMATICA function can be used, as is, for the input of another MATHEMATICA function. This makes it easy to use part of the output of a complicated expression in further calculations without the need to retype this part of the output.

MATHEMATICA has always been an excellent resource for the serious minded programmer. It offers programming in procedural, and functional languages among others. By using MATHEMATICA's programming language combined with its many built in functions it is possible to achieve a tremendous economy of programming compared with more traditional programming languages such as FORTRAN. For example in the tour guide section of the MATHEMATICA book (pages 1 - 21), no programmer could fail to be impressed by the efficiency of

```
RandomWalk[n_, d_] := NestList[(# + (-1)^Table[Random[Integer],
 {d}]) &, Table[0, {d}], n]
```

to code a random walk in arbitrary dimensions and then
`Show[Graphics3D[Line[RandomWalk[1000,3]]]]` to plot a 1000 step random walk in three dimensions.

MATHEMATICA is a wonderful resource for manipulating complicated mathematical functions such as regularized hypergeometric functions, elliptic functions and Airy

functions to name a few. For example did you know that

$$\frac{1}{2}\sqrt{\pi}z\text{MeijerG}\left[\{\{\},\{\}\},\{\{0\},\{\}\{-\frac{1}{2}\}\},\frac{z}{2},\frac{1}{2}\right]$$

is equivalent to `Sin[z]`?

The most significant new developments in MATHEMATICA 5 are its vastly improved numerical solving capabilities. There are excellent numerical tools for differential equations, linear programming and optimization problems. These numerical tools rival purpose developed numerical packages. Moreover they have wonderful error tracking capabilities.

The ability of MATHEMATICA to import and export in many different formats is another strong feature of this product. For example MATHEMATICA notebooks can readily be exported in `html` format for the World Wide Web.

There is only one aspect of MATHEMATICA 5 that I am disappointed with and that is its graphics capabilities. That is not to say that the graphics capabilities of MATHEMATICA 5 are not good. They continue to be impressive, but they are no longer cutting edge, and with a product that is in every other way cutting edge this is a disappointment. I was hoping to have graphics capabilities where I could interactively rotate 3D plots, and interactively change axis labels, colours etc., although some of this is possible on PC platforms or in UNIX (using the undocumented experimental RealTime3D, type `<< RealTime3D'` in your Mathematica session and you will be able to rotate 3D plots and even zoom in on them if you hold the control key down). Despite the limited interactive graphics capabilities, the user can make use of the extensive range of plotting options to readily produce high quality graphics suitable for publication.

In summary, MATHEMATICA 5 is a brilliant tool, no scientist or engineer whether in research, industry or education should be without access to it. The only product that comes close to it is MAPLE 9 and whether one is slightly better than the other depends as much on the user's personal style than it does on anything else. Both products have a great deal of research and development behind them and we are the beneficiaries of having two products of this calibre that can inspire one another to push the boundaries further. Neither of the products is likely to disappear off the shelves and MATHEMATICA 5 is increasingly shaping itself as the product of choice for industry. MATHEMATICA 5 is also an invaluable product in mathematics education. One of the things that is most important in mathematics education is practice. Mathematical expertise and mathematical insights are developed by working through large numbers of examples. MATHEMATICA 5 is a gold mine for creating examples and their solutions both algebraically and graphically. If you are a high school student about to head off to university to pursue science, maths, engineering, biology, finance ... then one of the best preparations you could provide yourself with is a student copy of MATHEMATICA 5 (or MAPLE 9) and a basic familiarity with its use. If you are a high school teacher I can't imagine how you could do without it. It's a product that I use everyday.

* MATHEMATICA 5 is available from Wolfram Research at US $1880 for the professional version; US $895 for the academic version; and US $139.95 for the student version.