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Sizing an Oval

Using the Pin and String Method

Garth Lien[1]

We begin by looking at the definition of an oval, or as it is more formally known, an ellipse. An *ellipse* is the set of all points (x, y) in the plane such that the sum of the distances from (x, y) to two fixed points is some constant. The two fixed points are called the *foci* (the plural of focus).

The line through the foci intersects the ellipse at two points, known as *vertices*, marked M_1 and M_2 in Figure 1. (Vertices is the plural of the term *vertex*.) This line segment joining the vertices is called the *major axis* and its midpoint is called the *center* (C) of the ellipse. The *minor axis* is the line segment perpendicular to the major axis which also goes through the center and touches the ellipse at two points, marked m_1 and m_2 in Figure 1.

The standard equation of an ellipse centered at the origin [the point $(0, 0)$] is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are some positive constants. The shape of a given ellipse will depend on the relationship between a and b.

Setting $y = 0$, we can see that the ellipse cuts the x-axis at $x = \pm a$. Similarly, the ellipse cuts the y-axis at $y = \pm b$. From this we can see that the length of the major axis is $2a$ and the length of the minor axis is $2b$.

If the center of the ellipse is (h, k), rather than the origin, then the equation of the ellipse becomes

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

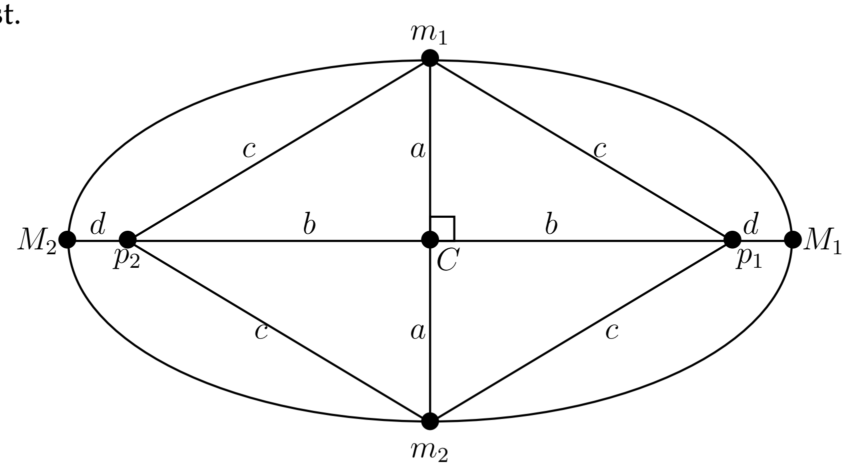
Thus setting the size of the ellipse in the equation is done by setting a and b appropriately.

It is relatively easy to geometrically construct an ellipse. In fact, sculptors, furniture makers and assorted craftsmen have been using ellipses in their designs for hundreds of years. Whilst the actual geometrical construction of the ellipse is simple, setting the exact dimensions requires accurate calculations. Fortunately these calculations only involve simple algebra, and so are quite accessible to both students and adults!

[1]Garth Lien is a *Parabola* reader from Wagga Wagga in NSW

The most common method for drawing ovals manually involves the use of two small nails (pins) set into the surface of material (such as wood, cardboard etc) and a loop of string slightly larger than the distance between these pins. When the loop of string is placed over the nails and stretched taught with a pencil, the oval can then be traced onto the surface by moving the pencil in a 360 degree arc over the material while keeping the string tight between the two pins and the pencil tip.

For example, suppose that you wish to make an oval jewelry box, and you want the oval to be an exact size, say $170mm$ wide by $300mm$ long. You could probably get an oval close to those dimensions by trial and error, but this would take a long time and could get a bit frustrating! Instead, you should look at the mathematics of the drawing method first.



When drawing an oval first set up the material by drawing a line down the middle lengthwise and by marking the centre on that line. From there each pin must be set into the surface of the material on each side of the centre at a specific distance along the long axis. Each pin should be equidistant from the centre, and the material should be cut slightly larger in length and width than the desired size of the oval. The pins cannot be set however, until the appropriate calculations are made.

Let P_1 and P_2 represent the points where we have to set our pins, and let C be the centre of our material. The distance between each pin and the center is b and the distance between the centre and the top of the oval is a. Since our loop of string fits over P_1 and P_2, and is always stretched tight when drawing, it must be the right length to make the pencil point hit all four maximum points exactly to make the oval the correct size. When the pencil is at M_1, our maximum right-hand point on the long axis, the loop of string is doubled onto itself. As the distance between P_2 and M_1 is given by $2b + d$, the loop of string is approximately $4b + 2d$ in circumference[2]. Also, when the pencil rests at m_1, our maximum upper point along the short axis, the loop hasn't changed length. Three points, both P_1 and P_2, as well as the pencil point mark the perimeter of the loop. Therefore, the loop can also be said to have circumference $2b + 2c$. We now have the equation

$$4b + 2d = 2b + 2c \quad \text{which implies} \quad b + d = c.$$

[2]The smaller the diameter of each pin, the more accurate this approximation will be.

Let ℓ be the distance from the centre of our oval to either of its maximums along the long axis. Then our hypotenuse c is equal to this distance as

$$2\ell = 2b + 2d \quad \text{or} \quad \ell = b + d = c.$$

Now we can calculate b since a and c are defined numerically. Using Pythagoras's Theorem we find $b = \sqrt{c^2 - a^2}$.

Finally we need to define the circumference of our loop algebraically. While the loop of string is $4b + 2d$ in length, we do not need to solve for d, since we already know ℓ. Let S represent the perimeter of our loop. Given $b + d = \ell$, we find

$$S = 4b + 2d = 2b + 2(b + d) = 2b + 2\ell.$$

Hence, applying this knowledge to our oval box above, each pin must be set out approximately $123.6mm$ from the centre, given

$$b = \sqrt{c^2 - a^2} = \sqrt{150^2 - 85^2} = 123.59.$$

Furthermore, the loop of string needs to be about $547.2mm$ in circumference since

$$S = 2b + 2\ell = 2(123.59) + 2(150) = 547.18.$$

With a few simple calculations, you can now make any size oval you want! Try it for yourself sometime!