Parabola Volume 42, Issue 2 (2006)

History of Mathematics: Classical Applied Mathematics

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Two separate events happily combined to suggest the topic for this issue's column. In the first place, I devoted my previous column to a somewhat controversial attempt to apply Mathematics to the 'softer sciences' such as Biology and Linguistics. It seemed to me that I really owed my readers a counterbalancing account of the resounding success of Mathematics as applied to the 'hard sciences' – Physics and Astronomy. But, as I was thinking along these lines, something else happened. I was asked to write a review of a recently published life of Laplace.

Laplace (Pierre-Simon Laplace, 1749-1827) was a French mathematician, who worked in Paris and who holds a highly-regarded place in the history of Mathematics. The book I was asked to review was *Laplace: A Determined Scientist*. Its author is Roger Hahn, an American historian who has made the life of this mathematician his own life-work. His study was published by Harvard University Press and it appeared last year. There have been other biographies, the best being that by C. C. Gillispie, R. Fox and I. Grattan- Guinness (*Pierre-Simon Laplace*, 1749–1827: *A Life in Exact Science*, Princeton, 1997).

It is interesting to compare these two different biographies. Hahn's concentrates on bringing his subject to life – the object is to leave us with a picture of what kind of a man he was. The earlier work, by contrast, focuses on his achievements and it tells in considerable technical detail about the mathematical investigations he undertook. The two accounts are thus complementary to one another; to form a well-rounded picture we need both.

In many ways, Laplace was a typical figure of an intellectual movement known as the Enlightenment. You will find a good general introduction to this in the online Wikipedia. Go to:

http://en.wikipedia.org/wiki/The_Enlightenment

For our purposes here, the pivotal figure was Isaac Newton (1643–1727). It was Newton who propounded the Law of Gravity, and so established a firm basis for theoretical Astronomy.

Prior to Newton, Astronomy was still regarded as an exact (mathematical) science, but the laws governing it were empirical. That Nature was regular and that the motions of heavenly bodies were periodic was known; these were already established by very precise observation, even in antiquity. What Newton did was to provide a theoretical framework within which these regularities could be explained in terms of fundamental laws.

It may seem strange to us now, but by no means were all scientists convinced at once by Newton. Although Laplace was not born until some 20 years after Newton

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died, it was still not sure then in everyone's minds that all the phenomena of Astronomy could be explained by Newtonian reasoning. The same was true of Mechanics, the operation of Newton's Laws in the terrestrial realm.

Laplace spent much of his working life demonstrating exactly the full scope and completeness of the Newtonian view of the universe. He was deeply committed to that view, and I titled my review of Hahn's book 'Newton's Bulldog'. This was to draw a parallel between the role that Laplace played in the promulgation of the Newtonian world-view and that played by Thomas Huxley ('Darwin's Bulldog') in the defence of Darwin's Theory of Evolution.

Thus Laplace was able to show that comets obeyed the same laws as did the planets and that the various satellites of those planets also obeyed those same laws as they in their turn orbited their planets. The regularity of the motions of heavenly bodies was, as mentioned above, known from antiquity. The motion of the sun provided clocks (e.g. in the form of sundials) and calendars based on changes in the appearance of the night sky as the seasons progressed. Calendars were also constructed based on the changing appearance of the moon. This precise periodicity was accepted as fact, but it was seen as lying outside the realm of the power of natural explanation.

Laplace showed that all of these regularities followed as natural consequences of Newton's Law of Gravitation. But it was believed by others that here was revealed the hand of God, a beneficent Providence keeping the solar system in order, and so revealing the power and goodness of the Creator. The question of the stability of the solar system, as this precise periodicity was termed, occupied Laplace greatly. He worked on the question on and off from 1773 to 1786, and finally announced success in the enterprise.

Actually he did rather less than demonstrate the exact periodicity of the entire solar system. (In fact, we now know that some minor bodies, not then discovered, do not obey strict periodicity; some asteroids, for example, have chaotic orbits.) However, he did sort out two important points. In the first place, he was able to show that what seemed to be contrary observations could be explained away as due to observational error. Secondly, he demonstrated that the miniature 'solar system' consisting of Jupiter and its four largest moons, did have a precise periodic solution, deducible entirely from Newtonian dynamics. He then supposed that what he had achieved in this particular case could be applied to the solar system in general.

This was seen as a major achievement, and it still stands as such. However, not everyone looked at it in this light. In 1802, no less a person than Napoleon Bonaparte asked Laplace where God found a place in his system of the world. Presumably he had in mind the still popular explanation of the stability as a manifestation of a benign Providence. But Laplace replied 'I have no need of that hypothesis'. The reply has become a famous one, and it has become a catch-cry of the discussion of the relation between Science and Religion, although it has been queried whether Laplace ever actually said it.

Hahn, however, is quite clear on the matter. (He is wonderfully thorough in his attention to even the tiniest details.) He writes, 'This oft-repeated phrase may not have been spoken verbatim, but [Sir William Herschel, the famous astronomer, who was present on the occasion] reports the gist of the exchange.'

[There is something of a parallel between this exchange and modern debates over the Theory of Evolution. Opponents of this theory often point to some phenomenon that is not yet entirely explained by it. This position, which is by no means typical of all religious stances, has come to be known by the denigratory term 'the God-of-the-gaps'. But the problem for it is that, as Science advances, those 'gaps' get smaller and smaller. (For more on this question, see the article under that heading in the online Wikipedia near the site detailed earlier.) As an example, when I was growing up, it was common to hear it said that the one thing known about the 'missing link' (between apes and humans) was that it was still missing! No informed person could possibly utter this sentence today. The position espoused by Napoleon, that one needs to invoke the power of God to explain the stability (i.e. periodicity) of the solar system, has now disappeared so completely that most people today have never even heard of it.]

So complete was the victory for Newtonian dynamics in this and other fields that the doubts over Newton's Laws disappeared completely. (They ultimately gave place to Einstein's Theory of General Relativity, but that is another story.)

Although the solar system is a very complicated structure, it is much simpler than the earth's ecosystem, and other such objects of biological study. We do use Mathematics to investigate the behaviour of biological systems, but we necessarily simplify them to make the process manageable. In fact, almost all applications of Mathematics to the real world involve some simplification. Perhaps the very simple Mathematics involved in balancing one's bank account or calculating the change in the supermarket is an exact replica of the real thing, but once we get beyond these, some simplification is involved.

More generally, we approach the mathematical analysis of a real world phenomenon by formulating a 'model'. A model is an imaginary system that is supposed to be like the real life one but amenable to precise (mathematical) analysis. If we refer to the real life situation as R, say, and the model as M, we are saying that:

R is like M.
So, if I can analyse and understand M,
Then I gain insight into R.

As a general rule, M will be simpler than R, as outlined above, but the hope is that the analogy will be good enough for the process to work. We hope our simplification is not an oversimplification.

It seems to me that this insight (that some simplification is needed) has never been better put than by the surrealist poet Corrado Costa. It takes the form of a movie review, the second in a suite of three, titled 'They go to see Three Movies'. This one is called 'The Life of Lenin'. Paul Vangelisiti's English translation reads as follows:

With absolute fidelity
the real time
of Lenin's life

is respected.
Reproduced with absolute fidelity
the dreams and insomnia
of Lenin. The crucial moments
of childhood, the school
years, everything is repeated, even random
conversations at a bus stop.
The silences are respected. The lapses.
The movie lasts 54 years.
You should see it
at least twice.

In a somewhat more mathematical vein is this observation taken from a recent biography of the pioneering biomathematician Pierre Verhulst: 'Mathematical modeling is a difficult art, akin perhaps to that of the caricaturist. We have to obtain a maximum of resemblance from a minimum [number of] features. We must know in advance to neglect what afterwards will [be seen to] be negligible.'

In the case of (say) the motion of a planet orbiting the sun, we can to a very good approximation ignore all the other bodies in the solar system, and so reduce the problem to the analysis of the two mutually gravitating bodies. We may even further simplify by regarding the bodies themselves as simple points, each endowed with an attached parameter called its mass. [This was demonstrated by Newton, and the simplification holds good as long as the planet and the sun are themselves composed of such 'particles' so arranged that their properties are spherically symmetric.]

It is indeed fortunate that these simplifying assumptions apply with great accuracy to the solar system. The sun and the earth, for example are very nearly spherical, and they are sufficiently distant from the rest of the solar system for these other bodies to be ignored. The one exception to this is the moon, which (fortunately!) is much smaller than either. Thus, it is possible to show that each planet follows an elliptical path about the sun, with only small deviations from this general rule. Furthermore, these small deviations may themselves be calculated from Newton's law as small perturbations from the strict elliptical orbits.

The success of the Newtonian of the world was so complete in fact that it was seen as possible to provide a complete picture of the entire universe, if one had all the data and the capacity to analyse it completely. Laplace wrote:

> 'An intelligence which, at any given instant, knew all the forces by which the natural world is moved and the position of each of its component parts, if as well it had the capacity to submit all these data to Mathematical analysis, would encompass in the same formula the movements of the largest bodies in the universe and those of the lightest atom; nothing would be uncertain for it, and the future, as also the past, would be present to its eyes.'

This passage has generated much debate. The 'intelligence' has earned for itself the name of the 'Laplace Demon'. The very possibility of such an intelligence entails a denial of the freedom of the will, which is a position that a few crackpots espouse, but which can hardly be taken seriously. [If a prisoner pleaded to the judge: 'The laws of Physics made me do it!', the excuse would hardly be taken seriously.]

You can see some of this debate by googling 'laplace demon'. When I tried this, the response gave 150,000 websites, so clearly the debate continues. However, the demon lost a lot of its force with the rise of Quantum Mechanics, and took a further knock with the discovery of chaos and its properties.

[An interesting sidelight on this debate is provided by the earlier responses, however. Several prominent mathematicians and physicists, in discussing the demon, came close to the positing of chaotic behaviour. I wrote about this in a journal of the history of Mathematics back in 1988. See my article 'Nineteenth Century Anticipations of Modern Theory of Dynamical Systems' in *Archives for History of Exact Sciences*, Vol. 39, pp. 183–194.]

The Newtonian picture is now known not to be exact in that extreme circumstances show some deviations from it. Some of these are covered by General Relativity, others by Quantum Mechanics. Nevertheless, its successes were enormous. In the language I used in the description above, the model M conformed to the reality R with remarkable fidelity. The fact that the Laplace demon was taken so seriously and led to such debate shows how sweeping was the success of the Newtonian model.