Parabola Volume 53, Issue 1 (2017)

Problems 1521–1530

Parabola would like to thank Sin Keong Tong for contributing Problem 1528. Problem 1523 was developed from a suggestion by Travis Dillon.

Q1521 Solve the equation

$$\sqrt{x+20}+\sqrt{x}=17\,.$$

Q1522 Consider a circle with centre $O=(a,b)$ and radius r, and a point $P=(p,q)$ which lies outside the circle. If Q and R are the two points on the circle such that PQ and PR are tangent to the circle, then find the equation of the line QR.

Q1523 Consider a list of the first n positive integers in some order; for example if $n=7$, then we could have $5,1,6,4,7,3,2$. We seek lists which have the following further property: for every integer k from 1 to n, the sum of the first k numbers in the list is a multiple of k. For instance, the above list **does not** have this property since the sum of the first 5 numbers is $5+1+6+4+7=23$, which is not a multiple of 5.

Find all lists with this property.

Q1524 Find all positive integers n such that

$$1122^{n-1}+2244^{n-1}$$

is a factor of

$$1122^n+2244^n\,.$$

Q1525 If the expression

$$(1-x)(1+2x)(1-3x)(1+4x)\cdots(1-(2n-1)x)(1+2nx)$$

is expanded and its terms are collected, then what is the coefficient of x^2?

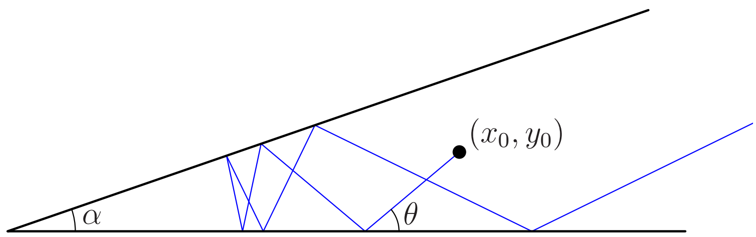
Q1526 Some questions about continued fractions – for basic information on these, see Peter Brown's article *Integer Points on Conics and Continued Fractions* in this issue.

(a) Find the continued fraction for $\sqrt{23}$.

(b) Find the continued fraction for $\frac{1}{3}\sqrt{11}$.

(c) Find the value of the continued fraction $[1;\,2,\,\overline{3,\,4}]$.

Q1527 In Problem 1517, we considered a billiard ball being projected into a "wedge–shaped table" as shown in the following diagram.



If the angle between the ball's initial trajectory and the horizontal is θ, and the angle at the vertex of the wedge is α, then how many times does the ball hit the wedge?

Q1528 The first question should be easy; the second harder; the third harder still!

(a) Find the coordinates of three points P_1, P_2, P_3 such that the distance between any two of them is one unit. If the point M is equidistant from P_1, P_2, P_3, then find the distance MP_1.

(b) Repeat as in part (a) but for four points.

(c) Repeat as in part (a) but for five points.

Q1529 For real numbers x, y, z such that

$$2x - 9y + 7z = 6 \quad \text{and} \quad 7x - 6y + 2z = 9\,,$$

evaluate $x^2 + y^2 - z^2$.

Q1530 Find all real numbers a, b, c such that $a < b < c$ and

$$a + b + c = 5\,, \quad a^2 + b^2 + c^2 = 15\,, \quad abc = 1\,.$$