Parabola Volume 54, Issue 2 (2018)

Problems 1561–1570

Parabola would like to thank Sin Keong Tong for contributing Problem 1563.

Q1561 Let a, b, c be positive numbers for which

$$\frac{a+b}{c} = 2018 \quad \text{and} \quad \frac{b+c}{a} = 2019\,.$$

Evaluate $\dfrac{a+c}{b}$.

Q1562 In a parallelogram $PQRS$, let M be the midpoint of PQ.
Find the cosine of $\angle RMS$ in terms of the lengths PM and PS and the angle $\angle MPS$.

Q1563 Given a positive integer n, add the digits of n; then add the digits of the result; and so on, until you obtain a one–digit number. This one–digit number is called the *terminating sum* of n.[1] Find the terminating sum for

$$n = 2018^{2017^{2016^{\cdot^{\cdot^{\cdot^{3^{2^{1}}}}}}}}\,.$$

Q1564 Write two numbers a, b in a row on a piece of paper. Form a list by writing their sum between them. Form another list by writing between every pair of adjacent numbers their sum. Repeat. For example, if $a = 1$ and $b = 2$, then we initially get

$$1\,, \quad 2\,.$$

our first list is then

$$1\,, \quad 3\,, \quad 2\,,$$

our second list is

$$1\,, \quad 4\,, \quad 3\,, \quad 5\,, \quad 2\,;$$

and so on. What is the sum of the numbers in the nth list?

Q1565 Two squares on a (normal 8×8) chessboard are said to be *neighbours* if they can be reached from one another by means of at most two horizontal/vertical steps, **or** at most one horizontal/vertical and one diagonal step. Find the maximum number of squares that can be chosen on a chessboard such that no two are neighbours.

Q1566 Let m and n be positive integers with $m \neq n$. Prove that $m^4 + 3n^4$ can be written as the sum of the squares of three non–zero integers.

[1] For more information on these sums, see the *Parabola* article Terminating Sum of Digits of a Positive Integer by Sin Keong Tong.

Q1567 Given a positive integer $n \geq 2$, find unequal real numbers a, b, **not** integers, such that

$$a - b\,,\ a^2 - b^2\,, \ldots,\ a^n - b^n$$

are all integers.

Q1568 Draw the graph of $\sin(y + |y|) = \sin(x + |x|)$.

Q1569 We have a row of n coins. A "move" consists of selecting a coin which is tails up, and turning over both that coin and the one (if any) immediately to its left. An example of a sequence of three moves involving five coins is

$$HTTTT \to HTTHH \to THTHH \to HHTHH\,.$$

Prove that if we are allowed to choose the initial arrangement of coins, then it is possible to make $\frac{1}{2}n(n+1)$ moves before getting stuck; but that it is never possible to make more than this many moves.

Q1570 Find all solutions of the simultaneous equations

$$2x = z(3x^2 + 3y)\,, \quad 2 = z(3x + 3y^2)\,, \quad x^3 + 3xy + y^3 = 5\,.$$