Parabola Volume 50, Issue 2 (2014)

Problems 1451–1460

Q1451 Use the ideas of the solution to problem 1443 (later this issue) to find *without calculus* the maximum value of

$$\frac{x}{(x^2+a^2)^2}\ ,$$

where a is a positive real number.

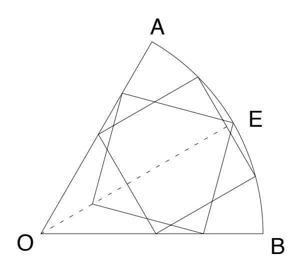
Q1452 As in problem 1442 (see the statement and solution later this issue), a particle is projected at a 45° angle from one corner of a 2014×1729 rectangle. Find the first occasion on which the particle hits the top wall and then the bottom, or the bottom and then the top, without hitting the left or right wall in between.

Q1453 In the town of truth–tellers and liars from problem 1444 (see the statement and solution later this issue), I meet four more people. I ask each of them, "How many of **the other three** are liars?"

George says "One". Helen says, "I don't know." Ian says, "Three." Jacqui says, "Two". Are these people truth–tellers or liars?

Q1454 If the roots of the equation $x^3+x+2014=0$ are $\tan\alpha,\ \tan\beta$ and $\tan\gamma$, can you evaluate $\tan(\alpha+\beta+\gamma)$? For a similar question see the solution to problem 1447, later this issue.

Q1455 The diagram shows two squares inscribed in a 60° sector of a circle; the points A, B and E are on the circular arc, and each square is symmetric about the angle–bisector OE. Prove that the squares have the same size.



Q1456 The positive integers a and b have no common factor. The positive integer n is a multiple of both a and b; exactly half the numbers from 1 to n are multiples of a or of b but not both. Find a and b.

Q1457 Mitchell and Dale are playing a game with dice. Mitchell has a die with five sides (each equally likely to show up) and Dale has a normal six–sided die. The two throw their dice alternately, with Mitchell going first. The first to throw a 1 wins. What are the winning chances of the two players?

Q1458 Add up the numbers $0, 1, 2, 3, 4, \ldots$ successively to get

$$0\,,\ 1\,,\ 3\,,\ 6\,,\ 10\,,\ 15\,,\ 21\,,\ 28\,,\ldots\,. \qquad (*)$$

The remainders when these numbers are divided by 8 are

$$0\,,\ 1\,,\ 3\,,\ 6\,,\ 2\,,\ 7\,,\ 5\,,\ 4\,,\ldots\,.$$

Notice that every possible remainder from 0 to 7 appears. For which numbers other than 8 is this true? That is: determine all positive integers m such that if we continue the sequence $(*)$ indefinitely and then find the remainder when each term is divided by m, all possible remainders from 0 to $m-1$ appear.

Q1459 It can be proved from the Peano axioms for arithmetic (see Michael Deakin's article in the previous issue) that every number except 1 is the successor of some number: that is, if $x \neq 1$ then $x = y^+$ for some y. Use this result, together with the axioms and the definitions of addition and multiplication (equations (1), (2), (8) and (9) in Michael's article) to prove the following.

(a) There are no numbers x, y such that $x + y = 1$.

(b) If $x * y = 1$ then $x = 1$ and $y = 1$.

Q1460 One more question about the coin–sharing problem (see the last two issues for the basic questions): what happens if there are 19 people sharing out 10 coins according to the same rules?