

Problems 1731–1740

Parabola would like to thank Les Gordon for contributing Problem 1735.

Q1731 Let $n = 1204$. The factors of n which lie between \sqrt{n} and n are

$$43, 86, 172, 301, 602,$$

and if we add these up we get our original number, $43 + 86 + 172 + 301 + 602 = 1204$. The same thing works for $n = 1316$. Find (without asking a computer to do it for you!) a number between 1204 and 1316 which has the same property.

Q1732 Suppose that the numbers a_1, a_2, \dots, a_n are equal to $1, 2, \dots, n$, but not necessarily in that order. Find the maximum possible value of

$$S = \sum_{k=1}^n (k - a_k)^2,$$

and the values of the numbers a_k which give this maximum.

Q1733 Alain is participating in a motor trial over a fixed distance, where each competitor is allocated a target time and has to drive at a fixed speed in order to reach the finish line exactly on time. Alain has his speed all worked out; but just as he is about to start, he is informed that his time allocation has been decreased by 10% because of financial irregularities by his support team. “No problem”, says Alain, “I’ll just increase my speed by 10%”. And so he did. And at the end of his allocated time, he was still some distance short of the finish. What went wrong?

Q1734 How many functions f from $\{1, 2, 3, 4, 5\}$ to $\{1, 2, \dots, 10\}$ satisfy the conditions

$$f(1) < f(2) \leq f(3) < f(4) \leq f(5) ?$$

Q1735 Let P be a point inside $\triangle ABC$; let AP, BP, CP meet the sides BC, CA, AB at the points D, E, F , respectively. Show that

$$\frac{|AE|}{|EC|} + \frac{|AF|}{|FB|} = \frac{|AP|}{|PD|}.$$

Q1736 If a polynomial $f(x)$ is divided by $x - a$, then the remainder is a constant r ; if $f(x)$ is divided by $x - b$, where $b \neq a$, then the remainder is s . If $f(x)$ is divided by $(x - a)(x - b)$, then the remainder will be a linear polynomial. Find it.

Q1737 Find all integers n for which $\sqrt{2024n + 1}$ is a positive integer.

Q1738 Find the smallest set of numbers S which has the properties

- 1 is in S ;
- if a, b are any numbers in S , then $1/(a + b)$ is also in S .

Q1739 A sequence is defined by $a_1 = 1, a_2 = m$ and

$$a_{n+1} = \frac{a_n^2 - 1}{a_{n-1}}$$

for each $n \geq 2$. Here, m is a fixed integer. Prove that every term a_n is an integer.

Q1740 Let a be an integer. Find the number of integers b for which the quadratic

$$(x + a)(x + b) + 2024$$

can be factorised as the product of two linear factors with integer coefficients.