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Editorial

Dear Readers

Welcome to this issue of *Parabola incorporating Function*. The issue starts with David Angell's article on solving a class of second order recurrences. Recurrence relations can look disarmingly simple because it is often a simple exercise to substitute in successive values to reveal the solution, a sequence comprised of these values. In solving recurrence relations you are seeking an algebraic expression that can be used to evaluate terms in the sequence directly, rather than recursively. In many cases it is not possible to find such an algebraic expression. Here is a rather famous example:

$$a(n) = \begin{cases} \frac{1}{2}a(n-1) & \text{if } a(n-1) \text{ is even,} \\ 3a(n-1)+1 & \text{if } a(n-1) \text{ is odd.} \end{cases}$$

If we start with $a(0) = 11$, which is odd so that $a(1) = 3 \times 11 + 1 = 34$, which is even so that $a(2) = \frac{1}{2}34 = 17$, which is odd so $a(3) = 3 \times 17 + 1 = 52$, which is even so that $a(4) = \frac{1}{2}52 = 26$, and then $a(5) = \frac{1}{2}26 = 13$ and $a(6) = 3 \times 13 + 1 = 40$, $a(7) = 20$, $a(8) = 10$, $a(9) = 5$, $a(10) = 16$, $a(11) = 8$, $a(12) = 4$, $a(13) = 2$, $a(14) = 1$. Indeed I am sure that if you start with a positive integer and generate the sequence from this recurrence relation you will always reach the number 1. Proving that this is true is not at all trivial. In fact no one has been able to prove it yet and it remains a conjecture, The Collatz Conjecture, named after Collatz who proposed it in 1937.

The second article in this issue, by Farid Haggar, also considers recurrence relations. In this article recurrence relations are used to obtain the solution to a variant of the classic monkey and coconut problem. The problem is complicated by the requirement that the solutions must be positive integers. This is an example of a so-called Diophantine problem, named after Diophantus of Alexandria. Farid's monkey and coconut problem is a variant of the classic problem written up by Ben Ames Williams in the 9 October 1926 issue of the *Saturday Evening Post*. The problem is also discussed at length in Martin Gardner's *The Colossal Book of Mathematics* (W.W. Norton and Company, 2006).

The third article, by Michael Deakin, continues his History of Mathematics series. In this article Michael considers factor analysis applied to test scores. The underlying idea in factor analysis is to determine if a number of observables $y_1, y_2, \ldots, y_n$ can be linearly related to a smaller number of unobservable factors $f_1, f_2, \ldots, f_k$. For example the observables might be scores from many different students in n different tests.

Please continue to send in solutions to problems or send in your own problems.

Editor

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