Parabola Volume 48, Issue 3 (2012)

Editorial

Dear Readers

As we go to press there has been a lot of excitement in mathematics about the proof of the ABC conjecture. The conjecture, which was posed by two prominent number theorists Joseph Oesterlé and David Masser in 1985, is considered to be one of the most important conjectures in number theory and the proof of the conjecture is a notoriously hard problem. It may be that the proof will be revealed to be invalid but the mathematican who came up with the proof, Sinichi Mochizuki, is very highly regarded so it may well be valid. Understandably it will take experts quite some time to wade through the 500 or so pages of proof.

So, what is the ABC conjecture? There are many different ways of stating the conjecture, some easier to understand than others. To begin, you might try the following exercise: take any two positive integers a and b having no common factor and add them to obtain a larger integer c; write down all the prime factors of a, b and c, ignoring any repetitions; multiply these primes to obtain a product P. The aim is to choose a and b in such a way that c is larger than P: you will usually find that this is not the case. Here are a few examples.

a	b	$c = a + b$	primes	P
2	3	5	$2, 3, 5$	30
3	4	7	$3, 2, 7$	42
8	27	35	$2, 3, 5, 7$	210
9	20	29	$3, 2, 5, 29$	870
64	11	75	$2, 11, 3, 5$	330

You may notice that we have tried to be increasingly clever in these examples: the only way that we could possibly make c larger than P is to give a, b and c lots of repeated prime factors, which will increase the size of c without affecting P (since each prime, no matter how often it occurs in c, counts only once in P). That's why we tried taking for a and b the numbers $8, 27, 9$ and 20, each of which has the same prime factor at least twice. In our last attempt we chose a and c first, with the same aim in mind. In these examples c is always smaller than P; however, if you persist, you may find an instance such as

$$100 + 243 = 343\,, \quad P = 2 \times 5 \times 3 \times 7 = 210$$

with $c > P$. So it is not *always* true that $c < P$. You can think of the ABC conjecture as stating that we can modify the inequality $c < P$ to obtain one that *is* always true by increasing the right-hand side in two ways: raising P to some power greater than 1, and multiplying by a constant. Thus:

The ABC Conjecture. If α is any number greater than 1, it is possible to find a constant K for which the following is true: for any positive integers a, b, c such that a, b have no common factor and $a + b = c$, we have $c < KP^\alpha$.

One of the interesting things about the conjecture is that it is not sufficient merely to multiply the right-hand side of the inequality $c < P$ by K. David Masser, mentioned above as one of the originators of the conjecture, has proved that there is no constant K for which c is always less than KP. The conjecture is also significant for its links with other difficult problems. For example, there are connections with Fermat's Last Theorem; some of them, amazingly, are actually quite simple. See if you can solve problem 1409 in this issue of *Parabola!*

For more about the ABC Conjecture, you might like to investigate the account in the article "Beyond the Last Theorem" by Dorian Goldfeld, published in *The Sciences* March/April (1996).

Editor

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