

Probability theory and wrongful conviction

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1 Introduction

Experts in mathematics and statistics usually create a model of reality to solve a real-world problem, and their model is often far simpler than reality itself. Therefore, their method is to start with a complicated, chaotic world and then move to a simplified world governed by the rules of mathematics and science. In the present article, we will go the other way, from a mathematical formula to chaotic reality, to determine the validity of a formula used in a famous legal trial. Here, we use Bayes' Theorem, but by using pictures, we have tried to make this article accessible to people without any knowledge of high school mathematics.

In 1984, a mathematical probability formula was presented as crucial evidence at a murder trial. The formula was the probability of having a specific blood type, and the defendant was convicted. The defendant served a ten-year prison term. Ten years had passed since his release from prison when the true culprit admitted to the crime.

The purpose of the present article is to study the formula presented at the trial and check its validity.

These days, they use DNA analysis instead of blood types, but the problem of using probability theory for the evidence remains crucial.

2 Historical background

On August 6th, 1949, the wife of a professor at Hirosaki Medical University was murdered. Police arrested Takashi Nasu. There were two vital pieces of evidence: one was the testimony by the victim's mother that Nasu looked very similar to the murderer, and another was that the blood type of the victim matched the blood type of the blood found on the shirt of Takashi Nasu. Although the victim's mother testified that Takashi Nasu looked very similar to the murderer, "looked similar" is very subjective evidence. On the other hand, the mathematically calculated probability of matching blood type was objective evidence. Or, at least, it seemed to be objective evidence.

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Prof. Tanemoto Furuhashi, a renowned expert in blood research at the University of Tokyo, considered the top university in Japan, conducted the blood type calculations.

His claim was as follows. The blood type of the victim was $BMQE$. More precisely, the type was B in the ABO system, M in the MN system, Q in the Qq system, and E in the Ee system.

Let the probability of finding a person with B , M , Q and E blood type be Y in Japan. It was known that $Y = 0.015$. The blood found in Takashi Nasu's shirt was of the same type.

According to Prof. Furuhashi, the probability that Takashi Nasu was a murderer was

$$W_1 = \frac{X}{X + Y} = \frac{1}{1 + 0.015} = 0.985222 \approx 98.5\%. \quad (1)$$

In the second trial held in 1952, he was convicted as a murderer and served 10 years.

Ten years had passed since his release from prison when the true culprit presented himself and admitted to the crime. Because of the statute of limitations, this true culprit was not arrested.

Takashi Nasu demanded a retrial, and Yomiuri News Paper and the Japan Federation of Bar Associations supported him. In 1977, the Sendai High Court declared that the trial in 1952 was a mistrial. The present article aims to answer the following questions.

Question 1 What is the meaning of Formula (1)?

Question 2 Is Formula (1) mathematically correct?

Question 3 Is Formula (1) responsible for the mistrial?

3 Some basic knowledge of probability theory

3.1 Conditional probability explained by pictures

We need Bayes' Theorem to understand Formula (1). This section's purpose is to prepare the reader for it. First, we calculate probabilities for a disease because the infection of a disease is a good example for the study of Bayes' Theorem.

Example 1. Consider some disease and a medical test to determine a person is infected with this disease. Suppose that

- (a) the percentage of infected persons is 0.1%;
- (b) the percentage of infected persons getting a positive test result is 99.9%; and
- (c) the percentage of non-infected persons who get a positive test result is 3%.

If a person tests positive on the test, what is the probability that this person has the disease?

To answer this question, we first define some events, as follows:

A is the event of infecting the disease;
 A^c is the event of not infecting the disease;
 B is the event that the test is positive; and
 B^c is the event that the test is negative.

Based on (a), the probabilities of the event A and the event A^c are 0.001 and 0.999. These events are denoted by a green rectangle and a blue rectangle in Figure 1. Note that the sizes of the event A and the event A^c are not drawn to scale in Figure 1, because the size of 0.001 is too small to draw in a picture.

Suppose that we have the event A . Then, based on (b), the probability of getting a positive test result is 0.99. Similarly, suppose that we have the event A^c . Then, based on (b), the probability of getting a positive test result is 0.03. These probabilities are also indicated in Figure 1.

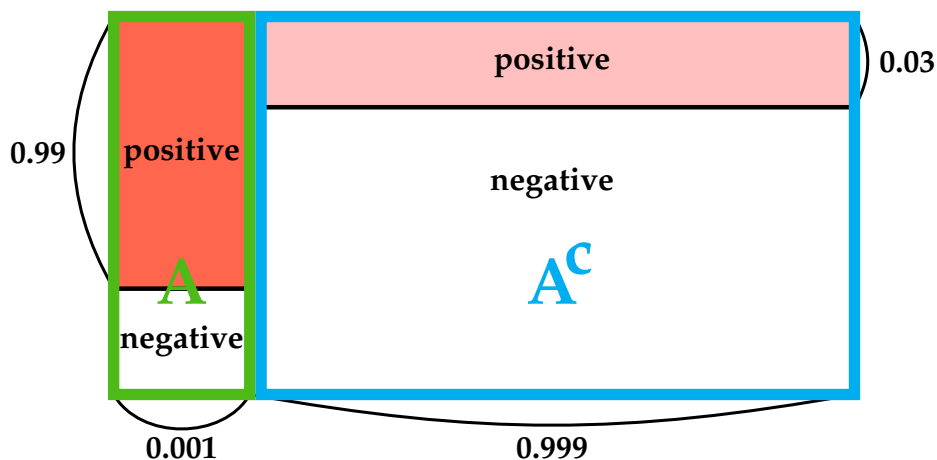


Figure 1: The probabilities of events A , A^c , B and B^c

Next, we calculate the probability of the event B . Event B of getting a positive test result consists of the red area and the pink area in Figure 1. Their total area is $0.999 \times 0.03 + 0.001 \times 0.99$. This is the probability of Event B .

Suppose we have the event B . We calculate the probability of getting infected. This is denoted by the red area, and the probability of being infected is the size the red area. This is 0.001×0.99 .

Therefore, if a person gets a positive test result on the test, then the probability that this person has the disease is

$$\frac{0.99 \times 0.001}{0.999 \times 0.03 + 0.001 \times 0.99} = 0.0319767 \approx 3.2\%.$$

These calculations are exactly what Bayes' Theorem asserts for this particular example.

3.2 Conditional probability explained by mathematical symbols

This subsection aims to explain Bayes's Theorem using mathematical symbols. Here, we presuppose the basic knowledge of conditional probability. You do not need this knowledge to read through this article, but learning to use mathematical symbols for probability theory is useful. These symbols are often used in economics, too.

Here, A and B are the events defined in Subsection 3.1. By the definition of conditional probability, we have

$$P(A \cap B) = P(A|B)P(B) \quad (2)$$

$$P(B \cap A) = P(B|A)P(A). \quad (3)$$

Since $P(A \cap B) = P(B \cap A)$, (2) and (3) imply that

$$P(A|B)P(B) = P(B|A)P(A).$$

Hence, if $P(B) \neq 0$, then

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}. \quad (4)$$

When a person tests positive on the test, the probability that this person is infected is $P(A|B)$. By (a), (b) and (c) in Subsection 3.1,

$$P(A) = 0.001, \quad (5)$$

$$P(A^c) = 0.999, \quad (6)$$

$$P(B|A) = 0.99 \quad (7)$$

$$\text{and} \quad P(B|A^c) = 0.03. \quad (8)$$

Hence, by (5), (6), (7) and (8) we have

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) = 0.001 \times 0.99 = 0.00099, \\ P(A^c \cap B) &= P(A^c)P(B|A^c) = 0.999 \times 0.03 = 0.02997 \end{aligned}$$

and

$$P(B) = P(A \cap B) + P(A^c \cap B) = 0.00099 + 0.02997 = 0.03096. \quad (9)$$

By (4), (5), (7) and (9),

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{0.99 \times 0.001}{0.03096} = 0.0319767. \quad (10)$$

This is Bayes' Theorem expressed as a mathematical formula.

4 How probability theory was used in the trial

With the knowledge we learned in Subsection 3.1, we are prepared to study the formula presented in the trial. As we saw in Section 2, in the second trial held in 1952, Takashi Nasu was convicted as a murderer and served 10 years in prison. In the trial, Prof. Furuhata's evidence was dominant in determining the verdict.

His evidence is based on Formula (11), where 0.015 is the probability of finding a person with *BMQE* blood type:

$$\frac{1}{1 + 0.015} = 0.985222. \quad (11)$$

Takashi Nasu served a ten-year prison term. After ten years since his release, the true culprit appeared in 1971. Takashi Nasu appealed to the court for a mistrial. After losing once, he won the case in 1977.

When Takashi Nasu was declared innocent, many people began to examine the evidence used in the trial in which he was convicted. At that time, Prof. Furuhata was ill and in hospital. Prof. Furuhata wrote that he had learned this formula from Prof. Yûsaku Komatsu, a renowned mathematician. If you look for him in the mathematical database zbMATH Open [8], then you will find that he published 137 articles in English, French and German, besides articles in Japanese, and more than 50 mathematics books. Prof. Komatsu was famous for his research on functional analysis but he also wrote some papers on probability theory. Therefore, Formula (11) should have been at least mathematically correct.

Prof. Furuhata and Prof. Komatsu kept silent during the trial and did not discuss the evidence. Some lawyers and law professors began to study Formula (1) while consulting mathematicians.

Many mathematicians thought that Formula (1) should have been replaced by the following Formula (12):

$$\frac{0.5}{0.5 \times 1 + 0.5 \times 0.015} = \frac{1}{1 + 0.015} = 0.985222. \quad (12)$$

Here, we use an argument similar to the one in Subsection 3.1. We define A as the event that the blood strain in Nasu's shirt is from the body of the victim, and hence A^c is the event that the blood strain is from someone other than the victim. Since we do not know the probabilities $P(A)$ and $P(A^c)$, we let $P(A) = P(A^c) = \frac{1}{2}$. See Figure 2. We want to calculate the probability that the blood strain is from the victim when the blood type of the blood strain is the same as the victim's blood type.

Then, by Figure 2 below, we have Formula (12).

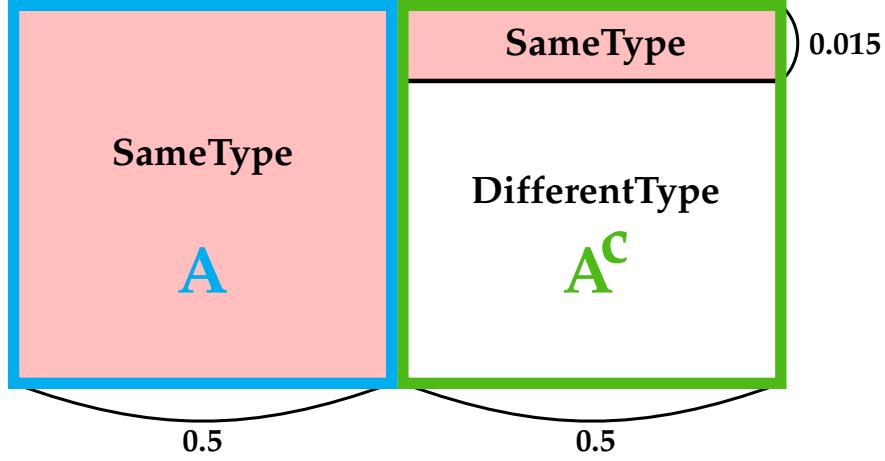


Figure 2: An illustration of Formula (12).

5 A criticism of the formula used in the trial

After the retrial of Takashi Nasu, legal scholars examined Formula (12) consulting mathematicians. Some legal scholars concluded Formula (12) was correct, and the probability of the event of Takashi Nasu was not a culprit was very small, but unfortunately, this event occurred.

Other legal scholars thought that there might be some flaws in Formula (12), and these flaws, combined with some unfortunate luck, caused the mistrial. In this section, we study Formula (12) to look for flaws.

5.1 Possibility of forgery of evidence by police

Legal scholars Prof. Norio Hamagami and Prof. Shigeru Kagayama proposed the following calculation in [5] as a substitute for Prof. Furuhashi's calculation. Here A is the event that the blood on Nasu's shirt was from the victim in the crime scene, B is the event that the blood on Nasu's shirt was from the victim's body stored in the police building, C is the event that the blood on Nasu's shirt was from someone with the same blood type other than the victim. We assume that $P(A) = 0.25$, $P(B) = 0.25$ and $P(C) = 0.5$. Then, we have the following table. You may be intrigued by the event B , but in the retrial of 1977, the Sendai High Court strongly suggested the possible forgery of evidence by the police. The police department in charge of this case did not admit the forgery of evidence, but the probability for police to forge the evidence always exists in every case. Therefore, it is natural to assume that $P(B) > 0$.

$$\frac{0.25}{0.5 + 0.5 \times 0.015} = 0.492611. \quad (13)$$

The probability that Nasu was the true culprit is relatively small in this calculation.

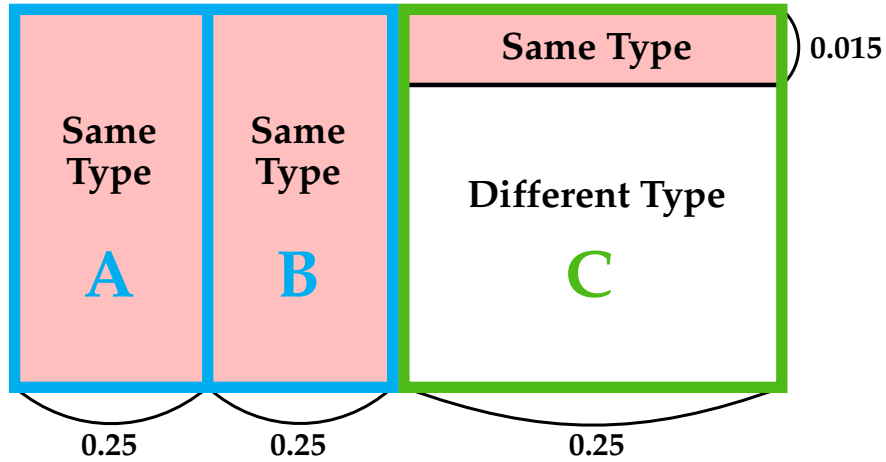


Figure 3:

6 Criticism of probability of having blood type $AMQE$

In Formula (12), the probability 0.015 is used. If the probability 0.015 is not correct, then Formula (12) is not correct as evidence.

In the following Subsection 6.1 and Subsection 6.2, we learn that using Qq type and Ee type was inappropriate as evidence in the trial. In Subsection 6.4, we learn that the probability of getting B depends on the prefecture.

Perhaps Prof. Noshiro, a mathematician, did not know about these facts when Prof. Furuhashi consulted him. Hence, it may be wrongful to criticise him for his advice to Prof. Furuhashi. However, we should know that applying a mathematical formula always risks presenting a wrong conclusion when we miss something important. He should have known that applying mathematics properly requires more than knowledge of mathematics.

Prof. Furuhashi used the following tables in the trial.

type	O	A	B	AB
%	30.5	38.3	21.7	9.5

type	M	N	MN
%	30	21	49

type	Q	q
%	33	67

type		E	e
%	O	100	0
%	A	82.4	17.6
%	B	87.0	13.0
%	AB	40.0	60.0

It is known that blood type ABO and MN are mutually independent. If we assume that ABO , MN and Qq blood types are mutually independent, then by using these

tables, the probability of having $BMQE$ blood type should be

$$0.217 \times 0.3 \times 0.33 \times 0.87 = 0.0186902 \approx 1.9\%$$

which is larger than the probability $0.015 = 1.5\%$ used as evidence. Note that the table for blood type Ee indicates that blood type Ee depends on blood type ABO . We do not know why Prof. Furuhashi used 0.015 instead of 0.0186902. Perhaps Prof. Noshiro was informed of the probability 0.015 and did not know about the above tables. However, the tables for blood types Q and E are meaningless, as you will now see.

6.1 On blood type Q

Prof. Furuhashi and Prof. Imamura claimed to have discovered the Qq blood type system in 1935; see [3]. He used this new blood type Qq to examine the evidence in the trial. However, in modern medical science, scientists agreed that the Qq system was the same as the P system. Even if the Qq system is the same as the P system, it is not improper to use Qq system in the trial, but the authors of the present paper discovered a fact that casts doubt on using the Qq system in the trial. In [4], Prof. Furuhashi and Prof. Imamura wrote about their experiment about P and Qq blood types. In their experiment, they used 38 people, and P blood type and Qq blood type coincide in 27 of them, but the two blood types did not coincide in 11 of them. How can we interpret this experiment? Although it is difficult to find whether the result of [4] is correct now, it seems that Prof. Furuhashi and Prof. Imamura's procedure to determine Qq blood type had some flaws.

6.2 On blood type E

Prof. Furuhashi used the blood type Ee to examine the evidence. Two Japanese medical researchers, Prof. Sutekichi Miyazaki and Prof. Naoji Sugishita, discovered the E blood type. However, this blood type is out of use in modern medicine because of the low success rate of determining it. The author could only find one reference for blood type E : see [2].

6.3 On the use of blood types Qq and Ee

Prof. Furuhashi was a leading medical scientist at the time of the trial and was eager to use the newest scientific knowledge for forensic medicine. It is natural for a scientist to be interested in newly discovered facts; however, it was not a proper attitude of a scientist to present newly discovered facts as evidence for a trial. Some things might not be known about the newly discovered facts. A scientist should use long-established scientific facts and procedures as evidence for a trial.

If we omit the use of blood types Q and E , then the percentage of BM type is 6.5%. Then, Prof. Furuhashi's original formula should be revised as follows:

$$\frac{1}{1 + 0.065} = 0.938879. \quad (14)$$

The probability 0.938879 is relatively small compared to the original probability 0.985. In particular, the originally estimated probability of Takashi Nasu being innocent was $1 - 0.985 = 0.015 = 1.5\%$. The revised probability of Takashi Nasu's innocence is $1 - 0.939 = 0.061 \approx 6\%$, which is a significantly higher probability.

6.4 Does the distribution of blood types depend on geography?

The distribution of blood types depends on geography. According to today's statistics, the percentage of Blood type B is 36.6% in Miyagi prefecture, 29.9% in Yamagata prefecture, 28.9% in Wakayama prefecture, and 28.3% in Aomori prefecture. These are prefectures where the percentages are high in Japan. On the other hand, it is only 6.1% in Tottori prefecture, 14.4% in Kagawa prefecture, 15.3% in Shiga prefecture, and 16.7% in Ishikawa prefecture. Prof. Furuhashi used 21.7% for the probability of having blood type B , but the incident occurred in Aomori prefecture, where the probability of having blood type B is 28.3%.

If we use 28.3%, then the percentage of BM type is 8.49%. Then, Prof. Furuhashi's original formula should be revised as follows:

$$\frac{1}{1 + 0.0849} = 0.921744. \quad (15)$$

The probability of $0.921744 \approx 92.2\%$ is quite small compared to the original probability of $0.985 = 98.5\%$, and even smaller than the probability given by the revised Formula (14).

Even if we dismiss the possibility of police forgery, Furuhashi's formula is very controversial. We want to add that blood type distributions vary between countries.

6.5 Possibility of error of inspection

Every forensic examination has a margin of error. For instance, a large-scale study [6] to determine the reliability of crime scene bloodstain pattern analysts, showed that, on samples with known causes, 11.2% of the analysts' conclusions were erroneous.

On the errors for defining blood type, we also discovered some important facts in [1]. When medical professionals determine blood types, they draw blood samples and mix them with antibodies to see how the red blood cells react. It is important to know this fact when we use blood type tests as evidence. If you know nothing about this procedure, then you may think that you can decide the blood type by looking at blood cells with a microscope. Instead, we use the reaction of red blood cells, and red cells may not react when the quantity is too small.

In some way, we must include the probability of errors in the formula used as evidence.

7 Emotions and judgement

Here, we study the emotional element of a trial. In [7], United States Supreme Court Justice Felix Frankfurter expressed doubts about judges' ability to submerge private feeling on every aspect of a case. As for the Formula (12), this was proposed by Prof. Furuhashi, an eminent professor of forensic medicine, and it was reasonable that the judge thought it very reliable.

At the time of the retrial, it was discovered that Prof. Furuhashi had not examined the blood on Nasu's shirt. He had ordered his graduate students to do that. Therefore, Prof. Furuhashi, who had a long experience of conducting an examination of blood, did not examine the evidence, and students without enough experience conducted the examination. The judge might not have trusted Prof. Furuhashi's opinion if the judge had known this fact.

Additionally, as we saw in Subsection 6.3, Prof. Furuhashi used his new research results on blood type Qq and blood type Ee as evidence. Since these results were not long-established facts of science, Prof. Furuhashi's contribution to the trial was misleading in two ways. One is that the judge trusted him because Prof. Furuhashi was an eminent scholar, and another is that he wanted to use his latest research results because he was then a leading scientist.

8 About the retrial

As we saw in a previous section, the true culprit revealed himself and Takashi Nasu won the retrial. The judge declared that this man was the true culprit because he knew the details of the murder incident that could be known to the true culprit. However, if he learned about the details of the incident from another person, then he might not be a true culprit. Because of the statute of limitations, this true culprit was not arrested. Therefore, his confession did not cost him much.

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