PROBLEMS

Most of these problems have appeared or been modified from exercises in a Russian school mathematics journal.

Q. 588. We are given a set of 201 different numbers with the property that the sum of any 100 of them is less than the sum of the remaining 101. Prove that all of the numbers are positive.

Q. 589. In a list of 200 numbers every one (except the end ones) is equal to the sum of the two adjacent numbers in the list. The sum of all the numbers is equal to the sum of the first 100 of them. Find that sum if the thirty fifth number in the list is -6.

Q. 590. S is a set of 7 positive integers whose product is equal to their sum. Find all possibilities for S.

Q. 591. Prove that if $|x| < 1$ and $|y| < 1$ then $|x + y| < |1 + xy|$.

Q. 592. Find the smallest positive multiple of 99999 which contains no 9 amongst its digits.

Q. 593. Given a computer which can produce the reciprocal of all available numbers and the sum of any two different ones. Which numbers can be produced if initially only the number 100 is in the computer, but new numbers may be stored in the computer's memory as they are calculated, and become available.

Q. 594. Show how to construct a triangle if one vertex, the centroid and the circumcentre are given.

Q. 595. The number 3 can be partitioned into positive integer parts in four different ways, if the order of the parts is regarded as significant. viz. $3 = 3$, $2 + 1$, $1 + 2$, or $1 + 1 + 1$. How many different ordered integer partitions exist for an arbitrary whole number n?

Q. 596. Prove that if all perfect squares are divided by 12, the remainder 8 is never obtained, but the remainder 9 is obtained in infinitely many cases.

Q. 597. If the lengths of the altitudes of a triangle are added in pairs, the three answers obtained are in the ratio 35 : 32 : 27. Find the largest angle of the triangle.
[Hint: First find the ratio of the sides of the triangle.]

Q. 598. i) If a_n is the nearest integer to $\sqrt{n}$, calculate

$$\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_{1984}} .$$

ii) If b_n is the nearest integer to $\sqrt[3]{n}$, calculate

$$\frac{1}{b_1^2} + \frac{1}{b_2^2} + \frac{1}{b_3^2} + \ldots + \frac{1}{b_{1984}^2} .$$

Q. 599. Bill Smith was a clear winner in the school chess tournament. Each of the twelve participants played one game with everyone else, scoring 1 point for a win, $\frac{1}{2}$ for a draw and 0 for a loss. Tom Jones performed well too; in fact he scored the same number of points as Alf, Bert, Colin, Dave and Ed altogether. What was the result of the game between Colin, who came eigth, and Jim Brown who finished seventh?

LATE SOLVERS: Andrew and Richard Stone of Canberra Grammar School sent independent solutions to the problems Q551, Q552, Q553, Q555, Q556, Q558, Q559, Q560, and Q562 as soon as possible after receiving their copies of Parabola, but too late for acknowledgement in the last issue. All answers were of a high standard, but Richard's solution of Q560, though correct as far as it went, stopped a little short of completeness.