# PROBLEM SECTION

It has been decided to revert to our initial policy of supplying solutions of the problems in the following issue. So two sets of solutions are given in this issue. If you are intending to send answers to some of the problems 684-695 whose solutions appear here earlier than expected, by all means still complete and send in your work. (For that matter if you have a fresh insight on any previous problem, you are always invited to share it with us.) But we would particularly welcome your attempts at the following probelms before the appearance of the second 1987 issue, which will contain the solutions.

Q. 696. $k$ is a whole number. There is a pile of $N$ coins shared amongst $n$ brigands as follows:

The first brigand takes 1 coin and $\frac{1}{k}$ th of the remainder.

The second brigand takes 2 coins and $\frac{1}{k}$ th of the remainder.

The third brigand takes 3 coins and $\frac{1}{k}$ th of the remainder.

⋮

The rth brigand takes $r$ coins and $\frac{1}{k}$ th of the remainder

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Finally the nth brigand takes the remaining $n$ coins.

Find $n$ and $N$ in terms of $k$.

Q. 697. I have a number $x$ which is divisible by 9 but not by 10. It has the property that when a certain one of its digits is deleted the result is the whole number $y = \frac{x}{9}$ which is also a multiple of 9. Prove that $y$ too has the property that when one of its digits is deleted it decreases 9 times. Find all possible values of $x$.

Q. 698. Find the smallest whole number $n$ such that $n^5 + 5^n$ is an exact multiple of 13.

Q. 699. Let $z = 0.1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14 \ldots$ where all the whole numbers are written consecutively after the decimal point.

a) Find the millionth digit after the decimal point.

b) Prove that $z$ is not a rational number.

Q. 700. The train line between X and Y is 666 kilometres. Every kilometre there is a post giving the distances from

X and Y :- 0|666, 1|665, 2|664, ...., 666|0.

Only one digit is used on the half way post:- 333|333. On how many of the posts are exactly two digits used?

Q. 701. Find all sets of integers $x, y, z$ such that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}.$$

Q. 702. Let $x$ and $y$ be positive integers with $y > 1$. Prove that it is impossible that

$$\underbrace{\sqrt{x + \sqrt{x + \sqrt{x + \ldots + \sqrt{x}}}}}_{y \text{ square roots}}$$

is a whole number.

Q. 703. All three sides of a right angled triangle are whole numbers of centimetres, the area is $A$ sq centimetres and the perimeter is $P$ centimetres. If $A = P$ find all possible side lengths.

Q. 704. Solve the equation

$$\sqrt{c - \sqrt{c + x}} = x.$$

Q. 705. We call an integer $n$ "good" if it can be represented in the form $5x + 11y$ where $x$ and $y$ are non-negative integers, and "bad" otherwise. (For example, all negative integers are bad, but 0 and $27 = 5 \times 1 + 11 \times 2$ are both good.)

Find a whole number $c$ with the property that whenever $c$ is expressed as the sum of two integers, one of them is good and the other bad. How many positive bad numbers are there?

Generalize both parts when 5 and 11 are replaced by $h$ and $k$, any two relatively prime whole numbers.

Q. 706. The following expression is expanded out, and all the coefficients of the resulting polynomial are added together. What result is obtained?

$$(1 - 3x + 4x^2)^{12}(5 - x - 3x^3)^{10}.$$

Q. 707. It is known that

$$2x_1 - 5x_2 + 3x_3 > 0$$
$$2x_2 - 5x_3 + 3x_4 > 0$$
$$\dots$$
$$2x_{n-1} - 5x_{n-1} + 3x_n > 0$$
$$2x_{n-1} - 5x_n + 3x_1 > 0$$
$$2x_n - 5x_{n+1} + 3x_2 > 0.$$

Let $x_1 = 7$. Find the numbers $x_2, x_3, \dots, x_n$.

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