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AMPLIFICATION OF TSUNAMIS IN COASTAL REGIONS

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Tsunami is the (Japanese) name given to a very long wave caused by an underwater earthquake or landslide. They used to be called tidal waves, but this is an inappropriate name because tsunamis have nothing to do with tides.

In this article we show how to estimate the value of the amplitude of a tsunami in a coastal region given the amplitude in the deep ocean. The strong amplification of a tsunami in shallow water is the major cause of destruction and flooding in coastal areas where tsunami are common.

Tsunamis progressing in the deep sea have a sea level variation given mathematically by

$$h = A\cos 2\pi(x/L - t/T) \tag{1}$$

where x is the distance from some fixed point at time t. (If you fix t you should be able to see that the shape of the curve looks like a wave, and if you draw the figure for different values of t you see it moving along.) Here h is the instantaneous sea level displacement, A is the wave amplitude, L is the wavelength and T is the period. This wave travels in water of depth D at speed

$$C = L/T = \sqrt{gD}$$

The first equality is obvious, the second is the outcome of a difficult derivation from the laws of hydrodynamics; g is the gravitational acceleration. On approaching a continental shelf of depth D_s the period of the wave is unchanged, however, the speed and the wavelength are reduced to the values C_s and L_s given by

$$C_s = L_s/T = \sqrt{gD_s}$$

The ratio of wavelengths for the wave on the continental shelf and in the deep ocean is therefore

$$L_s/L = \sqrt{D_s/D} \tag{2}$$

Now the energy in (one metre width of) a wave can be shown to be

$$E = 1/2 \rho A^2 L$$

where ρ is the density of water, so that if no energy is lost or reflected on approaching the shelf [not an accurate approximation, but one which will give some reasonable indication of the upper values of tidal variation to be expected on the shelf], equating energy of the wave in the deep ocean and on the shelf gives

$$A_s^2 L_s = A^2 L$$

The ratio of amplitudes is therefore

$$A_s/A = \sqrt{L/L_s} \tag{3}$$

Using (2) in (3), the ratio of amplitudes is given in terms of the depths as

$$A_s/A = \sqrt[4]{D/D_s}$$

For example, if $D_s = 16m$ and $D = 4096m$ $D/D_s = 256$ and $\sqrt[4]{256} = 4$. Hence $A_s/A = 4$ and a tsunami having an amplitude of 2m in the deep ocean will have an amplitude of 8m. (26.2ft) in water of depth 16m.

This wave would inundate low lying areas. Since its amplitude is almost equal to half the depth, it would probably have broken by the time it reached the beach, causing much damage to coastal development.

Note that, in this simple model, the ratio of depths is important. The wave period T , the speeds of propagation, and the wavelengths are not relevant.