Parabola Volume 62 Issue 1 (2026), Article 4

Beyond the classical straightedge: Exploring the potential of the finite double-edged straightedge

Fabian Schmitt[1] and **Christopher C. Tisdell**[2]

1 Introduction

Tension between the *theory* of constructive geometry and the *practice* of constructive geometry has existed for thousands of years [9]. This is partly due to the mismatch between the geometrical moves that are assumed to be theoretically possible, and the physical limitations of the tools that are manufactured and used. For instance, consider the contrast between the theory of producing straight lines, and the design of the straightedge tool (a.k.a., the unmarked straightedge) to produce such a line. In the learning, teaching and practicing of geometric constructions, the theory tells us that *any* two points can be joined together to form a line segment, ray, or line. However, the straightedge that we hold in our hand cannot be unbounded as it is a finite, manufactured tool! Thus, there are situations where we may be given a pair of points whose distance apart is greater than the side of our straightedge - a wicked problem that many have struggled with at one time or another [9].

Take a moment to picture the straightedge you use in your geometry work (or even in casual drawing). If you have one nearby, then pick it up and look at it closely. The tool in your hand may be classified as a straightedge due to certain markings thereon (which are ignored in classical, constructive geometry), but it probably has a similar *shape* as Figure 1, which we believe is the most common form of the straightedges that have been designed, manufactured and used in geometrical constructions over many years.

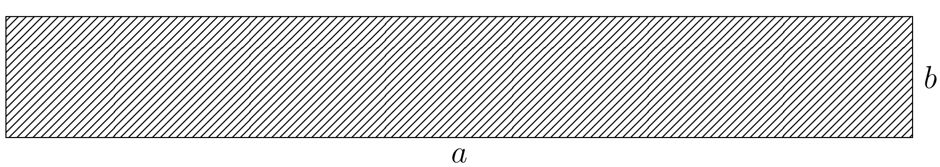


Figure 1: A finite double-edged straightedge

But, look closer! The tool in Figure 1 has more than one straightedge! Indeed, observe that there are *two* pairs of *parallel* straightedges in Figure 1: a longer pair and a shorter pair. The sides intersect to form corners. What if we consider using two

[1]Fabian Schmitt is a student at the Technical University Berlin, Germany
[2]Christopher C. Tisdell is a mathematics professor at UNSW, Australia

straightedges (i.e., the longer, parallel edges) in Figure 1 as a way to perform constructive geometry? Such a conceptual tool has been termed as "a double-edged straightedge" or "a parallel straightedge" by Birrel [1] and Yates [11]. However, the finiteness of our physical tool is in contrast to Birrel and Yates who made assumptions of infinite length of two of the parallel sides.

In this paper, we explore the possibilities for using the *finite double-edged straightedge* at its full potential, and explore several different opportunities to resolve some of the tension between theory and practice in the learning, teaching and practising of constructive geometry. In particular, this extra feature will mean that we can remove the use of a compass, a tool that has been shown to have many problems in the practice of geometry; see [8].

2 Operations with straightedges

Throughout the history of geometry, mathematicians have introduced many different tools and ways to use them [2, 4, 12]. The most common tool in use in schools today is a combination of the marked straightedge and the double-edged straightedge: a straightedge which is shaped like a basic rectangle. This combination also enables the assumed operations of the *finite double-edged straightedge*. Let us explore some of these tools, to find out what this tool can do.

2.1 The "classical" straightedge

The "classical" straightedge, also called *Euclidean ruler*, is only used to draw a line through any two given points and has no marks on it [4]. Therefore, there are two operations allowed for "classical" straightedge [3]:

(a) To place the edge of the straightedge in coincidence with a point;

(b) To draw a straight line.

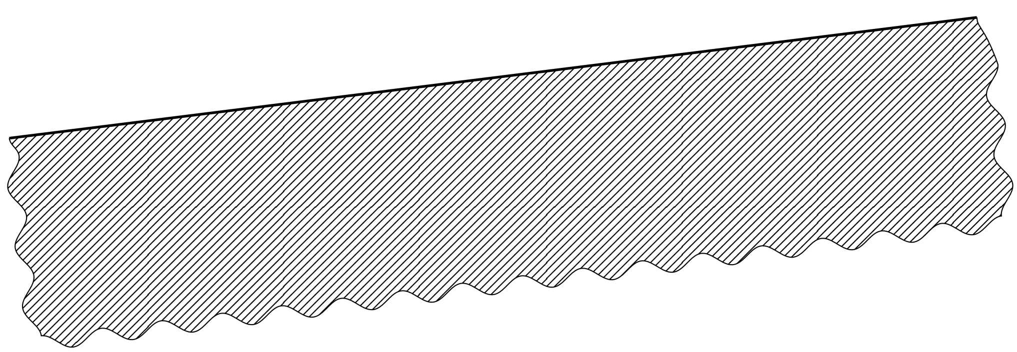


Figure 2: The "classical" straightedge

Aligning the straightedge with a line or two points is just the composition of (a). These two operations are the foundation for all other more specific straightedges.

2.2 The double-edged straightedge

The double-edged straightedge with parallel sides is a composition from two straightedges in a certain distance to each other. Therefore, this tool can create parallel lines [1]. The allowed operations of this new tool are:

(a) To place the edge of the double-edged straightedge in coincidence with a point;

(b) To draw a straight line.

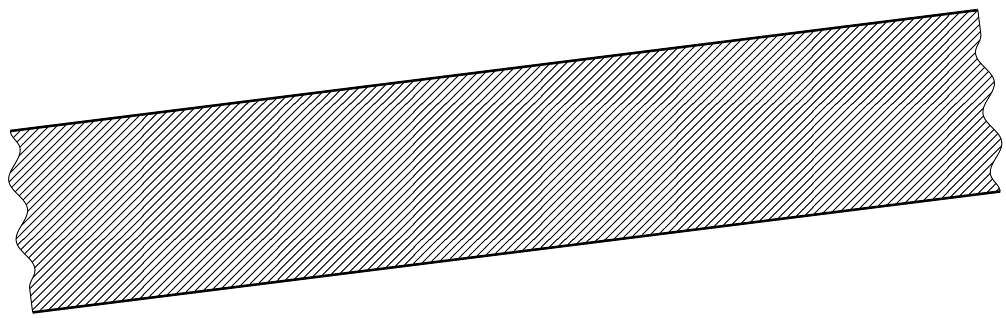


Figure 3: The double-edged straightedge

As you probably noticed, the operations are identical with the operations of the "classical" straightedge, but the second operation can be used in two ways: either draw it on the one or the other side of the straightedge, hence creating a set of parallel lines.

2.3 The marked straightedge

The marked straightedge is a "classical" straightedge with two marks R and S on its edge [4]. Therefore, you can mark off unit segments and new operations are possible.

(c) To align one marked point on the straightedge with a given point;

(d) To draw a line along its edge with the length $|RS|$;

(e) To rotate around the fixed point R until the second mark S coincides with a line.

Using the marked straightedge like (e), it becomes apparent that the marked straightedge is not only a straightedge, but a replacement for a fixed compass with radius $|RS|$ as well [4].

2.4 The finite double-edged straightedge

The *finite double-edged straightedge* is an abstraction of the double-edged straightedge, because it is finite. Its shape is rectangular, with side lengths a and b with $a > b$. The finite length can be used to mark distances, just as the marked straightedge could. Therefore, the following operations are allowed for this type of straightedge, which is a good resemblance to most straightedges we use:

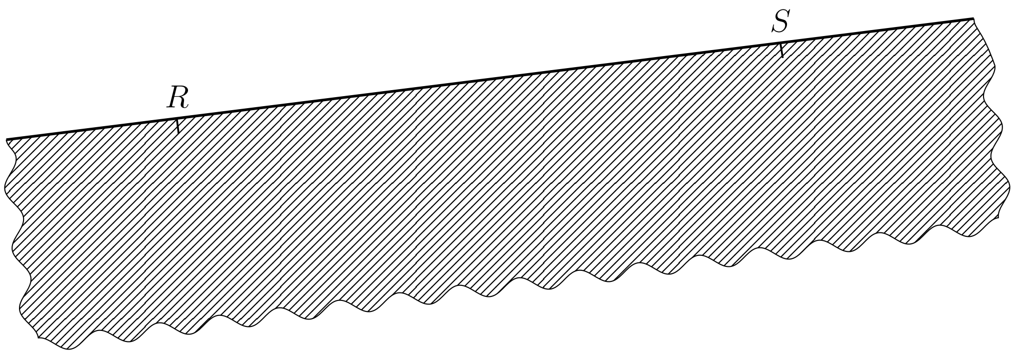


Figure 4: The marked straightedge

(a) To place the edge in coincidence with a point;

(b) To draw a straight line;

(c) To align one corner of the straightedge with a given point;

(d) To draw a line along its edge with the length a or b;

(e) To rotate the straightedge around one of its corner until another corner coincides with a line.

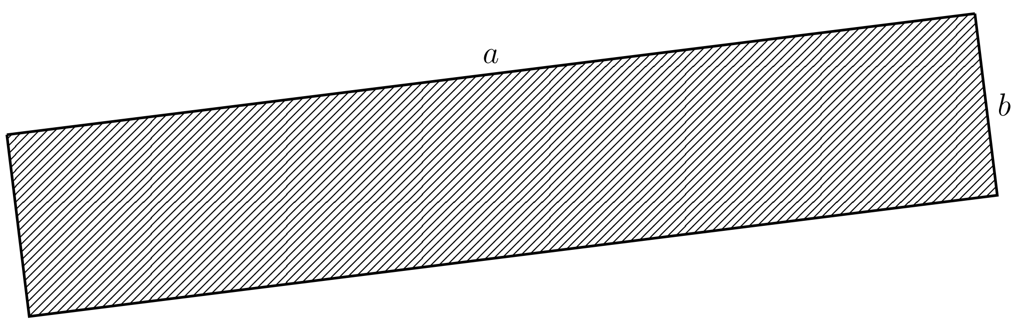


Figure 5: The finite double-edged straightedge

An infinite line can be created as well by drawing a line and shifting the straightedge to extend it.

An important characteristic of the *finite double-edged straightedge* is that its longer side is significantly longer than its shorter side, so $a \gg b$ is given.

3 Solutions

Let A and B be two points with $|AB| > a > b$ where a and b are the lengths of the two sides of the *finite double-edged straightedge*. Then directly joining them with the *finite double-edged straightedge* is impossible.

The three presented solutions are based on a certain geometric theorem and named after them. The goal of each is to construct a point Z which is collinear with A and B.

3.1 Pappus' Theorem

The solution using Pappus' Theorem was presented in [6].

3.1.1 Construction

The construction can be performed in the following steps:

1. Draw 2 arbitrary rays from A.
2. Choose 2 points C and D on them with $|CB| < a$ and $|DB| < a$.
3. Draw lines through C and B as well as D and B.
4. Choose E on DB and F on CB such that E and F lie outside of the "inner area" between the rays in which B lies.
5. Extend the lines EC and FD until they intersect AC in H and AD in G.
6. Extend EH and FG to get the intersection Z.
7. Connect Z with B and extend this connection.

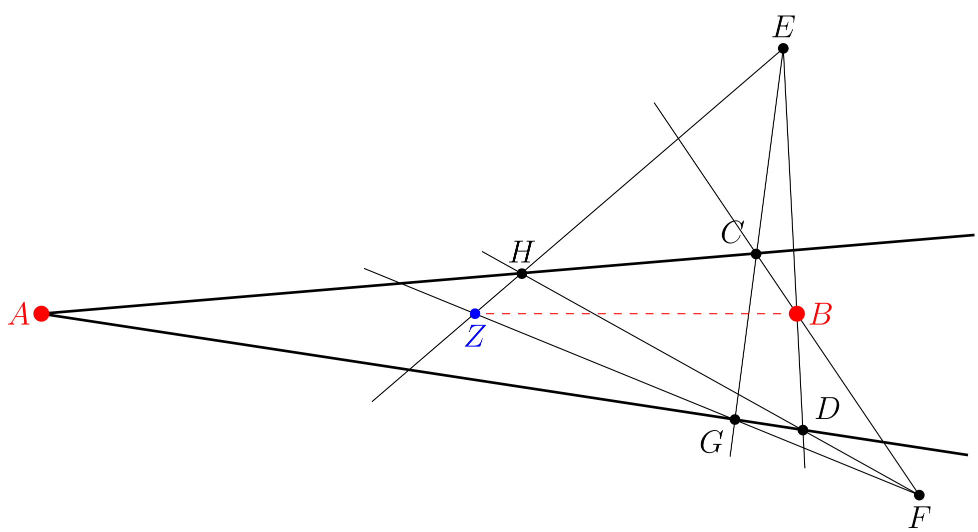


Figure 6: The construction using Pappus' Hexagon Theorem

If the second step cannot be performed, then redo step one and draw rays closer to B until the second criteria is met.

Then Z is collinear with A and B if $|AZ| < a$ or $|ZB| < a$, and the connection can be extended to connect A and B. This construction relies on $|EH| < a$ and $|FG| < a$ as well as Z to be close to either a or b to work, and in case one of these conditions is violated, either E or F have to be chosen again or the algorithm has to be performed again between either A and Z or B and Z.

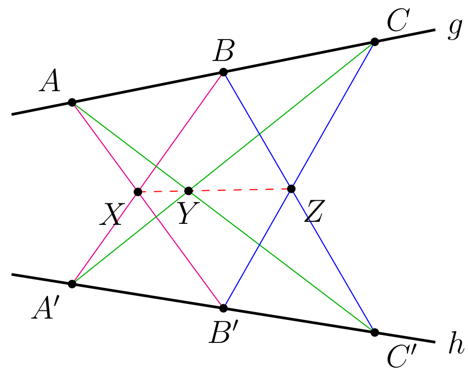
3.1.2 Proof

To prove that Z is collinear with A and B, Pappus' Theorem [5, p. 4] and its dual form [7, p. 409] will be used.

Theorem 1 (Pappus' Hexagon Theorem)**.**
Let two lines g and h be given. If $A, B, C \in g$ and $A', B', C' \in h$, then the points

$$X = AB' \cap BA'$$
$$Y = AC' \cap CA'$$
$$Z = BC' \cap CB'$$

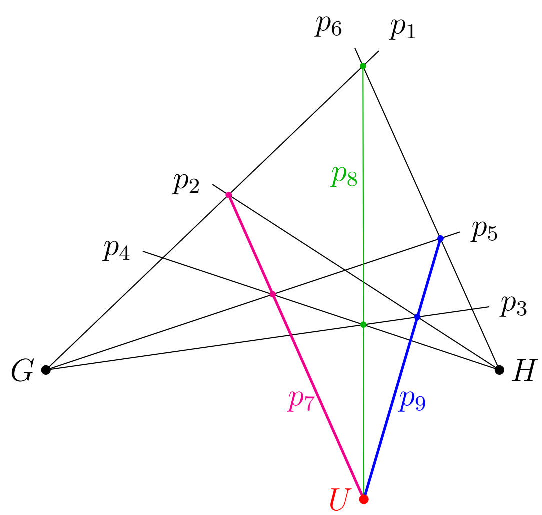
are collinear.



Theorem 2 (Dual Pappus Hexagon Theorem)**.**
If six lines p_1, p_2, p_3, p_4, p_6, p_6 are chosen alternately from two pencils with centres G, H, then the lines

$$p_7 := (p_1 \cap p_2)(p_4 \cap p_5)$$
$$p_8 := (p_6 \cap p_1)(p_3 \cap p_4)$$
$$p_9 := (p_2 \cap p_3)(p_5 \cap p_6)$$

are concurrent; i.e., they have a point U in common.



Proof. By construction, according to Figure 6, the points C and D are chosen such that $|CB| < a$ and $|DB| < a$. Let E be a point on line DB and F a point on line CB such that both lie outside the sector determined by the two rays through A.

Consider the three lines through E: ED, EG, EZ and the three lines through F: FC, FH, FZ, where $G = AD \cap EF$, $H = AC \cap EF$ and $Z = EG \cap FH$. Each of these pairs of lines intersects the lines AC, AD and AB, respectively.

Thus, the six lines are alternately chosen from the pencils with centres E and F. By the Dual Pappus Hexagon Theorem (Theorem 2), these three pairs of intersections are concurrent. The common point of concurrency is A. Therefore, Z must lie collinear with A and B, as required. □

3.2 Desargues' Theorem

The solution using Desargues' Theorem was presented in [6] as well.

3.2.1 Construction

The construction can be carried out using the following steps:

1. Draw 2 arbitrary rays from A.
2. Choose 2 points C and D on them with $|CB| < a$ and $|DB| < a$.
3. Draw a line through C and D and choose a point E on this line outside of the "inner area" between the rays.
4. Connect BC and BD and extend this connection.
5. Draw an arbitrary line q from E with $H := q \cap DB$ and $I := q \cap CB$.
6. Draw an arbitrary line r from E with $G := r \cap AC$ and $F := r \cap AD$.
7. Draw the connections GI and FH and find their intersection $Z := GI \cap FH$.
8. Connect Z with B and extend this connection.

If the second step is impossible, then redo step one and draw close lines until the second step can be carried out.

If $|ZB| > a$, then repeat this algorithm or choose different positions of F, E, C and D. This construction requires $|CB| < a$, $|DB| < a$, $|CD| < a$, $|GI| < a$, $|FH| < a$

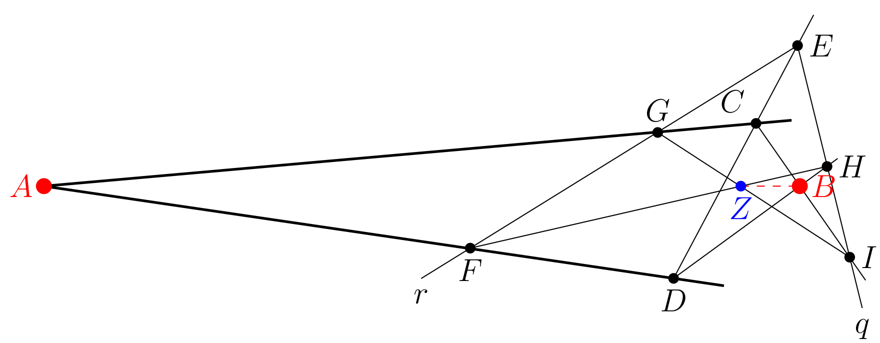


Figure 7: The construction using Desargues' Theorem

and $|ZB| < a$. Satisfying these conditions is fairly easy compared to the construction via Pappus' Theorem, if the points C, D, E and F are chosen in a radius of b around B. Therefore, all points can be joined using the longer side a of the *finite double-edged straightedge*.

3.2.2 Proof

To prove that Z is collinear with A and B, Desargues' Theorem [5, p. 271] will be used.

Theorem 3 (Desargues' Theorem)**.**
The connecting lines of the couples of corresponding vertices of two triangles $\triangle ABC$ and $\triangle A'B'C'$ are intersecting at a point S if and only if the intersection points of the couples of corresponding sides $P = BC \cap B'C'$, $Q = AC \cap A'C'$, $R = AB \cap A'B'$ lie on a line s.

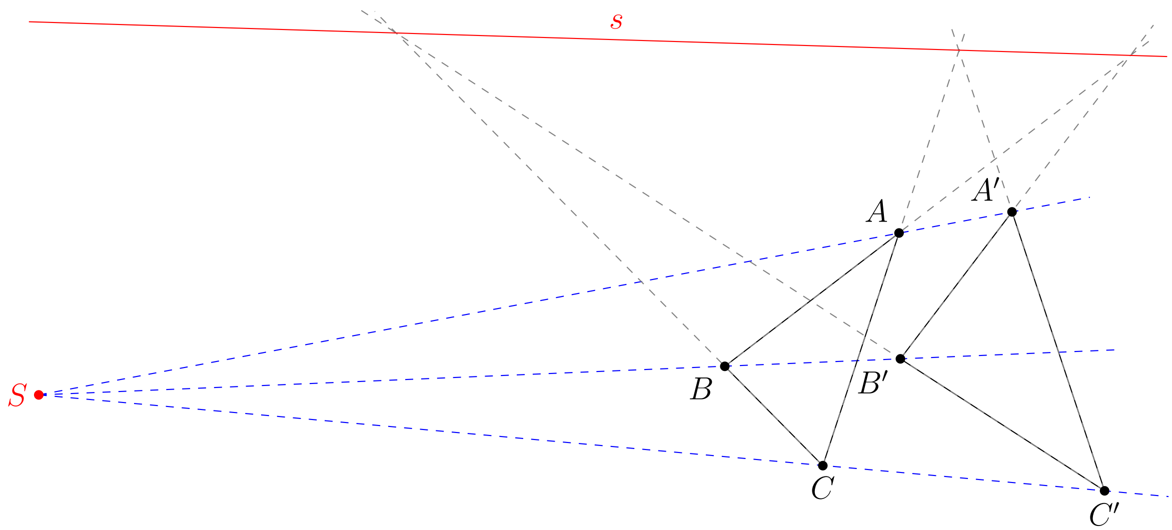


Figure 8: Desargues' Theorem

Proof. From the construction, we obtain two triangles $\triangle GFZ$ and $\triangle CDB$. The corresponding sides of these triangles intersect as follows:

$$GF \cap CD \in q\,, \quad FZ \cap DB \in q\,, \quad ZG \cap BC \in q\,,$$

where q is the auxiliary line through E.

Hence, the triangles $\triangle GFZ$ and $\triangle CDB$ are in axial perspective with axis q. By Desargues's Theorem (Theorem 3), it follows that they must also be in central perspective. Thus, their corresponding vertices are concurrent at a point. In particular, G lies on AC, F lies on AD, and Z must align with A and B. Therefore, A, B, and Z are collinear. □

3.3 Pascal's Theorem

You will probably have noticed that the first two constructions are possible with a mere finite one-sided straightedge. This next construction will use the properties of the *finite double-edged straightedge* fully, especially by rotating it around its corner to mark points at a distance b.

3.3.1 Construction

The construction using Pascal's Theorem can be performed using these steps:

1. Draw 2 arbitrary rays from A.
2. Find the intersections E, F, G, H of the rays with a circle with radius b around B.
3. Connect E, F with B and extend this connection.
4. Determine the intersections I and J with a circle with radius b around B.

5. Connect and extend I and H as well as G and J.
6. Find the intersection Z of these extended connections.
7. Connect and extend B with Z.

The second and the forth step can be performed by rotating the *finite double-edged straightedge* around its corner and marking all points in a distance b as soon as the corner of the shorter side contacts a line. In most cases, in which the arbitrary rays are

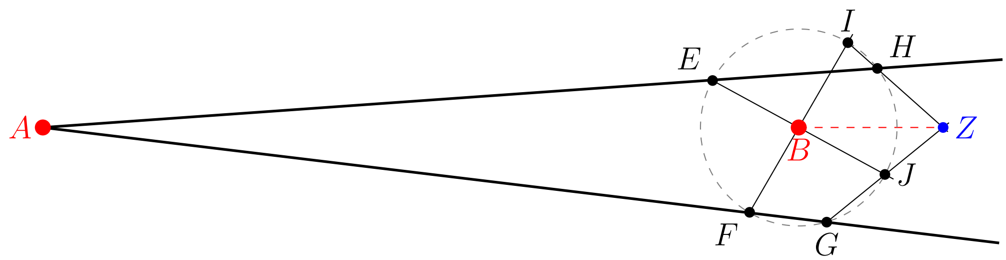


Figure 9: The construction using Pascal's Theorem

close to B, the connection can then be drawn. In case the rays are rather tangential, distinguishing between the points becomes increasingly difficult and point B cannot be joined to point Z. Therefore, redraw the rays in that case. This case is illustrated in Figures 10 and 11, in which only the construction around B is depicted. From Fig-

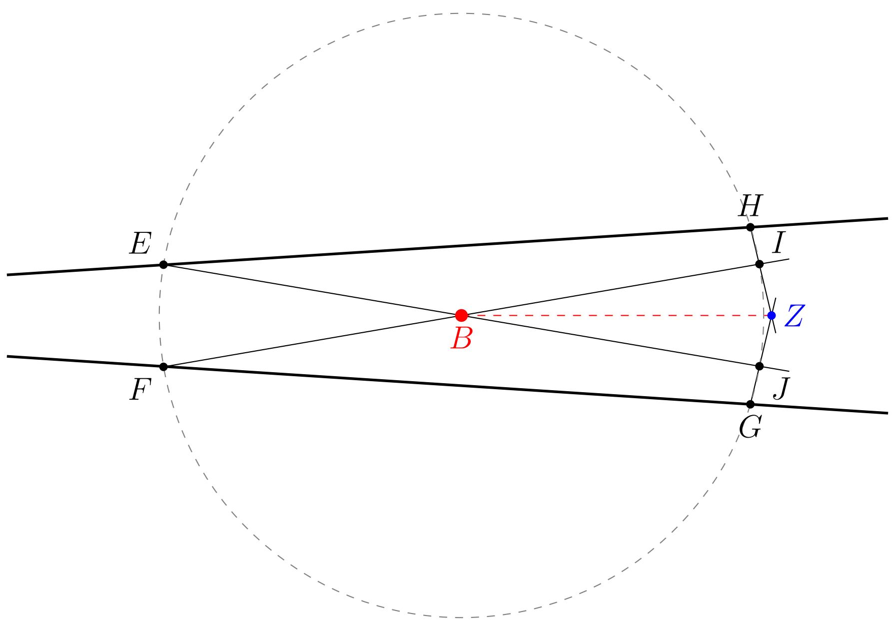


Figure 10: Rays close to B

ures 10 and 11, it can be deduced that either drawing the rays too close to B or too far away from B makes the construction more difficult. In the second case, one may not be able to join B and Z, whereas the first construction is complicated due to possible inaccuracies while determining I and J.

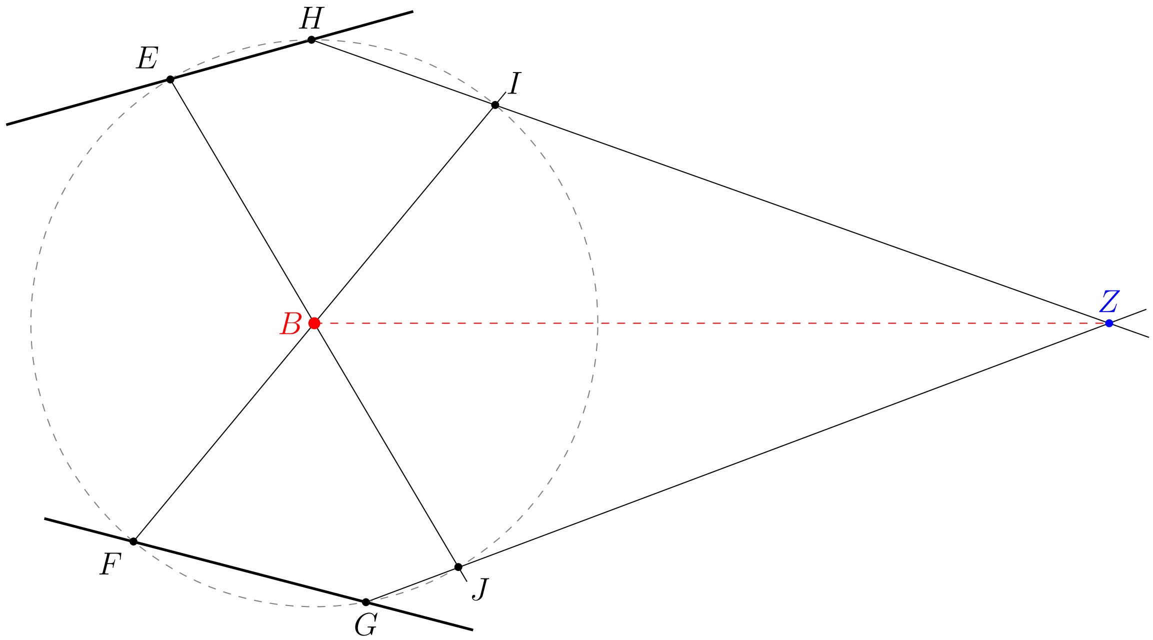


Figure 11: Rays nearly tangential

3.3.2 Proof

The proof of validity for this construction relies on Pascal's Theorem [5, p. 184].

Theorem 4 (Pascal's Theorem)**.**
A hexagon $AB'CA'BC'$ is inscribed in a conic section k if and only if the points $P = AB' \cap A'B$, $Q = B'C \cap BC'$, $R = CA' \cap C'A$ are collinear.

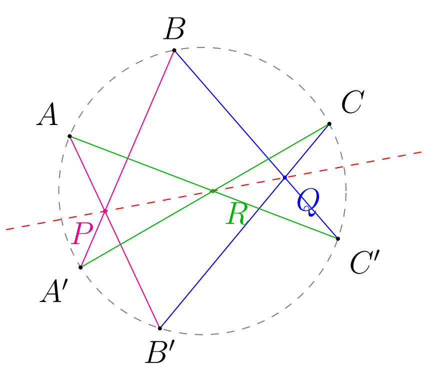


Figure 12: Pascal's Theorem

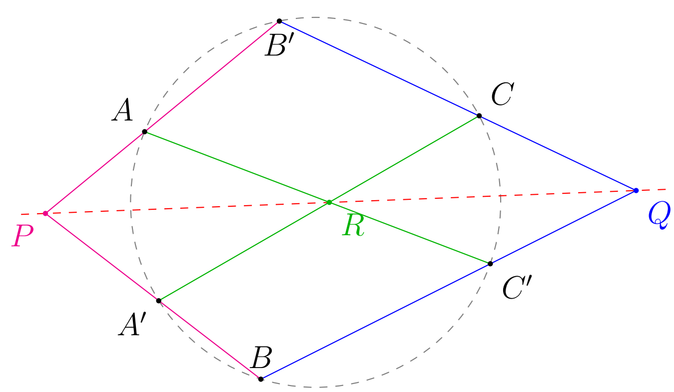


Figure 13: Pascal's Theorem with switched indices

For this construction, the conic section k will always be a circle, and the important cases are displayed in the Figures 12 and 13. For this problem, a construction similar to Figure 13 will be used.

Proof. In the construction, points E, F, G, H, I, J are defined as intersections of the rays from A and the circle with radius b centered at B. By construction, the line EH meets the line FG at A, and the line EJ meets the line FI at B.

Since all six points E, F, G, H, I, J lie on the circle with radius b, they form a hexagon inscribed in a conic. By Pascal's Theorem (Theorem 4), the intersection points of opposite sides of this hexagon are collinear. In particular, the intersection $Z = (EH \cap FJ)$ lies on the line determined by A and B. Thus, Z is collinear with A and B, as required. □

3.4 Other methods

Different solutions to this problem were discussed by [10] and [9]. These methods use a finite one-edged straightedge and a fixed compass. The method introduced by Yanagihara [10] works by bisecting the distance AB until the new constructed point can be joined with either A or B. The method presented in [9] is more efficient and uses the intercept theorem to correctly construct a point which can be joined with B to solve the problem.

It can be argued that the *finite double-edged straightedge* can perform all operations needed to construct both methods because of its similarities to the compass, but these methods are way less simple compared to the presented constructions.

4 Comparison of the solutions

Looking at the three different solutions to this problem, it becomes obvious that some of them are "simpler" or constructible in fewer steps. Thus, clearly defining the operations is key.

4.1 Geometrography

The geometrography of operations in geometry has not been defined for the *finite double-edged straightedge*. Thus, a comparison with the classical operations is the crucial to define them reasonably.

4.1.1 Geometrography of classical constructions

According to [3], for the traditional straightedge and compass setup, the operations in Table 1 below are considered. Placing the edge of the straightedge in coincidence with two points is $2R_1$. Therefore, every geometric construction with these tools can be represented by

$$l_1R_1 + l_2R_2 + m_1C_1 + m_2C_2 + m_3C_3\,.$$

with l_i and m_i representing the amount of times a certain operation is performed. With these coefficients, the *simplicity* of a construction can be defined as $l_1+l_2+m_1+m_2+m_3$ and the *exactitude* as $l_1 + m_1 + m_2$.

To place the edge of the straightedge in coincidence with a point	R_1
To draw a straight line	R_2
To put a point of the compasses on a determinate point	C_1
To put a point of the compasses on an indeterminate point of a line	C_2
To describe a circle	C_3

Table 1: Operations of the traditional setup

4.1.2 Geometrography of the *finite double-edged straightedge*

As mentioned, the "value" of operations of the *finite double-edged straightedge* are undefined in literature, but its operations are quite similar to the combination of a straightedge and a compass.

It is possible, to place this straightedge in coincidence with one or two points, and to draw a straight line. These operations are identical to the "classical" straightedge. Aligning one corner of the *finite double-edged straightedge* with a point is similar to putting one point of the compass on a determinate (or indeterminate) point. To rotate the straightedge around its corner until another corner coincides with a line is similar to describing a circle with a compass. Therefore, the following operations will be abstracted from [3] and adapted to this problem:

To place the edge of the straightedge in coincidence with a point	R_1
To place the edge of the straightedge in coincidence with two points	$2R_1$
To draw a straight line	R_2
To put a corner on a determinate point	R_3
To rotate the *finite double-edged straightedge* around its corner	R_4

Table 2: Operations of the *finite double-edged straightedge*

Therefore, the construction can be described by

$$l_1R_1 + l_2R_2 + l_3R_3 + l_4R_4$$

with the *simplicity* of the construction as $l_1 + l_2 + l_3 + l_4$ and the *exactitude* as $l_1 + l_3$.

4.1.3 Geometrography of the constructions

With clearly defined operations, it is possible to compare the three constructions, assuming that the optimal case is achieved by being able to join A and B after one iteration. The construction using Pappus' Theorem can be described by

$$16R_1 + 9R_2 + 0R_3 + 0R_4\,,$$

the construction using Desargues' Theorem can be described by

$$16R_1 + 10R_2 + 0R_3 + 0R_4$$

and the construction using Pascal's Theorem can be described by

$$13R_1 + 7R_2 + 2R_3 + 2R_4 \,.$$

Hence, the *simplicities* and *exactitudes* from Table 3 can be achieved.

Construction	*Simplicity*	*Exactitude*
Pappus' Theorem	25	16
Desargues' Theorem	26	16
Pascal's Theorem	24	15

Table 3: Comparison of the different methods

Comparison Comparing the three methods, with this chosen definition of *simplicity* and *exactitude*, the solution we obtained using Pascal's Theorem is the simplest and most exact solution. This relies on our definition of exactitude and simplicity, and it is reasonable to argue that rotating the *finite double-edged straightedge* around its corner cannot be compared to describing a circle or should be included weighted differently.

Nonetheless, this solution has a higher chance of producing a collinear point, which can be joined with B, due to less variables and less choices - only the rays are random. The following construction is completely determined. The two other constructions require more choices which influence the location of the collinear point drastically.

On balance, our method using Pascal's Theorem is not only simpler and more exact compared to the two other methods but also producing overall more consistent collinear points. This algorithm uses the allowed operations of the *finite double-edged straightedge* fully. Thus, an algorithm was found which compensates for the biggest weakness of the *finite double-edged straightedge*: not being able to connect two points which are too far apart.

5 Conclusion

As shown in this article, it is possible to connect two points A and B using an uncommon tool for standard geometry - the finite double-edged straightedge. Of the three presented constructions, the new method using Pascal's Theorem is the *simplest* and most *efficient*. It can be argued that this construction is more difficult to perform, because a real straightedge may have slightly rounded corners which may distort the construction. It would be interesting to see how well these different methods perform using physical straightedges. Nonetheless, the construction using Pascal's Theorem produces the collinear point with the highest reliability. Therefore, in most cases there is no need to redo the construction.

This article has presented simple solutions and compared the constructions, and the different advantages and disadvantages of each method were discussed. It is important to notice that, even though geometry is such an old scientific topic, new questions continue to arise and new answers can still be found.

Let us leave you with some questions. In Figure 1, we assumed that the side with length a is longer than the side with length b. Is there an optimal ratio $a : b$ in the design of the finite double-edged straightedge? Is there a ratio that will optimize the work of geometric constructions? What are the limits in terms of its shape?

Acknowledgements

This work was supported, in part, by Research Internships in Science and Engineering (RISE) Worldwide through the German Academic Exchange Service (DAAD), funded by the German Federal Ministry of Research, Technology and Space.

References

[1] S.K. Birrell, *Euclidean Constructions: Alternative Tools To The Traditional Compass And Straightedge*, PhD thesis, California State University, Northridge, 1983.

[2] G.C. Hunaus, *Die geometrischen Instrumente der gesammten praktischen Geometrie*, C. Rumpler, Hannover, 1864.

[3] J.S. Mackay, The geometrography of Euclid's problems, *Proceedings of the Edinburgh Mathematical Society* **12** (1893), 2–16.

[4] G.E. Martin, *Geometric Constructions*, Springer New York, NY, 1997.

[5] J. Richter-Gebert, *Perspectives on Projective Geometry: A Guided Tour Through Real and Complex Geometry*, Springer, Heidelberg, 2010.

[6] StackExchange, How can we draw a line between two distant points using a finite-length straightedge?, last accessed on 2025-08-25.

[7] S. Tabachnikov, *Projective Configuration Theorems: Old Wine into New Wineskins*, Springer, 2019.

[8] C.C. Tisdell and D.B. Olmedo, Beyond the compass: Exploring geometric constructions via a circle arc template and a straightedge, *STEM Education* **2(1)** (2022), 1–36.

[9] F.D. Trinh and C.C. Tisdell, Help! My straightedge is too short!, *Parabola* **60 (3)** (2024).

[10] K. Yanagihara, On some methods of construction in elementary geometry, *Tohoku Mathematical Journal, First Series* **16** (1919), 41–49.

[11] R.C. Yates, The angle ruler, the marked ruler and the carpenter's square, *National Mathematics Magazine* **15(2)** (1940), 61–73.

[12] R.C. Yates, *Tools: A Mathematical Sketch and Model Book*, Louisana State University, 1941.