Parabola Volume 62 Issue 1 (2026), Article 15

Problems 1791–1800

Problems 1792 and 1793 are based on suggestions kindly contributed by Prof. Marcel Chiriţă, Bucharest, Romania.

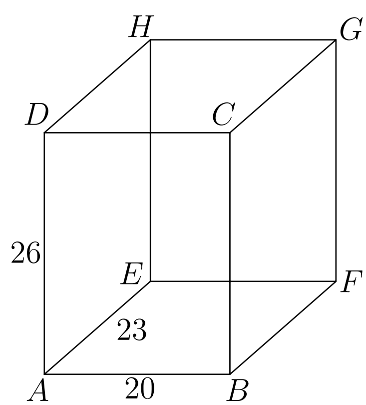
Q1791 Consider an "old–fashioned" 12–hour clock face having an hour hand, minute hand and second hand. Is there any time at which the three hands are equally spaced around the clock face? That is, any time at which each pair of hands is separated by $120°$? If so, then find all such times.

Q1792 Let x be a real number with $0 < x < 1$, and let $p = 1 - x$ and $q = \sqrt{4x - 3x^2}$. Prove that $3^p + 3^q > 4$.

Q1793 Let $n \geq 3$. Three different numbers are chosen at random from $\{ 1, 2, \ldots, n \}$; what is the probability that they are in arithmetic progression?

How does this change if we delete the word "different"? In other words, what is the probability if choosing the same number more than once is possible?

Q1794 (a) Let $ABCDEFGH$ be a rectangular prism having side lengths $AB = 20$, $AE = 23$ and $AD = 26$. Taking AB, AE, AD to be the x, y, z axes respectively, a particle is projected from A along the line with equation $x = y = z$. There is a small aperture for the particle to escape at each of the vertices. Which vertex will the particle escape from?



(b) Can you find positive integers a, b, c to replace the dimensions $20, 23, 26$ in the previous part, in such a way that the particle exits from A?

Q1795 We have a sequence of numbers $a_1, a_2, \ldots, a_n$. Each a_j is equal to 1 or 2, and the sum of all a_j is an even number $2A$. There is **no** sequence of consecutive numbers $a_p, a_{p+1}, \ldots, a_q$ with sum A. Find all possibilities for the original sequence.

Q1796 There is a pile of 2026 stones. Two players alternate in removing and throwing away stones so as to leave a smaller pile according to the rule: from n stones, one can leave either $n-1$ or $\lfloor n/2 \rfloor$ stones. Here, $\lfloor n/2 \rfloor$ means $n/2$ rounded to the next integer downwards if n is odd: for example, from a pile of 123 stones the player can leave either 122 or 61. Whoever removes the last stone wins. If players always make the best possible moves, then does the first or second player win?

Q1797 There are a number of towns, some of which are connected by roads. Prove that, if there are $2n$ towns which are the endpoints of an odd number of roads, then one can make n separate trips which, between them, travel along all the roads exactly once each.

Q1798 Let $D = \{\,0, 1, 2, 3, 4\,\}$ be the set of base–5 digits. We wish to count sequences $d_1 d_2 \cdots d_n$ of n digits from D which satisfy the following condition:

$$\text{if } k = 1, 2, \ldots, n-1 \text{ and } d_k \neq 4, \text{ then } d_{k+1} > d_k.$$

That is, sequences in which every digit (other than the last) is succeeded by a larger digit, except that 4 may be succeeded by any digit. How many such sequences are there if $n = 10$?

Q1799 (a) Prove that, if the ratio of the side lengths in a rectangle is rational, then the rectangle can be cut into a collection of congruent isosceles triangles.

(b) We now explore the converse of (a). That is: if a rectangle can be cut into congruent isosceles triangles, then does it necessarily follow that the ratio of side lengths of the rectangle is rational? This will involve a number of questions over the next few issues of *Parabola*, so first we ask the following.

Prove that, if a rectangle can be cut into congruent isosceles **right–angled** triangles, then the ratio of side lengths of the rectangle is rational.

Q1800 Let x, y, z be positive real numbers such that $xyz = 1$. Show that

$$\frac{x^2}{3+(x-1)^2} + \frac{y^2}{3+(y-1)^2} + \frac{z^2}{3+(z-1)^2} \geq 1\,.$$