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Proof without words: Triple-angle formulae for sine and cosine

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Theorem 1 (Triple-angle formulae)**.** *As shown in Figure 1, for $\triangle BDE$,*

$$\cos 3\theta = \frac{|ED|}{|BD|} = \frac{2\cos\theta\sin\theta(1-4\sin^2\theta)}{2\sin\theta} = \cos\theta(1-4(1-\cos^2\theta)) = 4\cos^3\theta - 3\cos\theta\,;$$
$$\sin 3\theta = \frac{|BE|}{|BD|} = \frac{1-(1-4\sin^2\theta)(1-2\sin^2\theta)}{2\sin\theta} = \frac{6\sin^2\theta - 8\sin^4\theta}{2\sin\theta} = 3\sin\theta - 4\sin^3\theta\,.$$

Several previous works [1, 2, 3, 4, 5] also present wordless proofs of these identities.

References

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[2] S. Kise, T. Uehara and T. Shinzato, Trigonometric ratios can prove the Pythagorean theorem, *Parabola* **61(3)** (2025), Article 12.

[3] R.B. Nelsen, *Proofs Without Words: Exercises in Visual Thinking*, Mathematical Association of America, Providence, RI, 1993.

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[5] S. Okuda. Proof without words: The triple-angle formulas for sine and cosine, *Mathematics Magazine* **74** (2001), 135.

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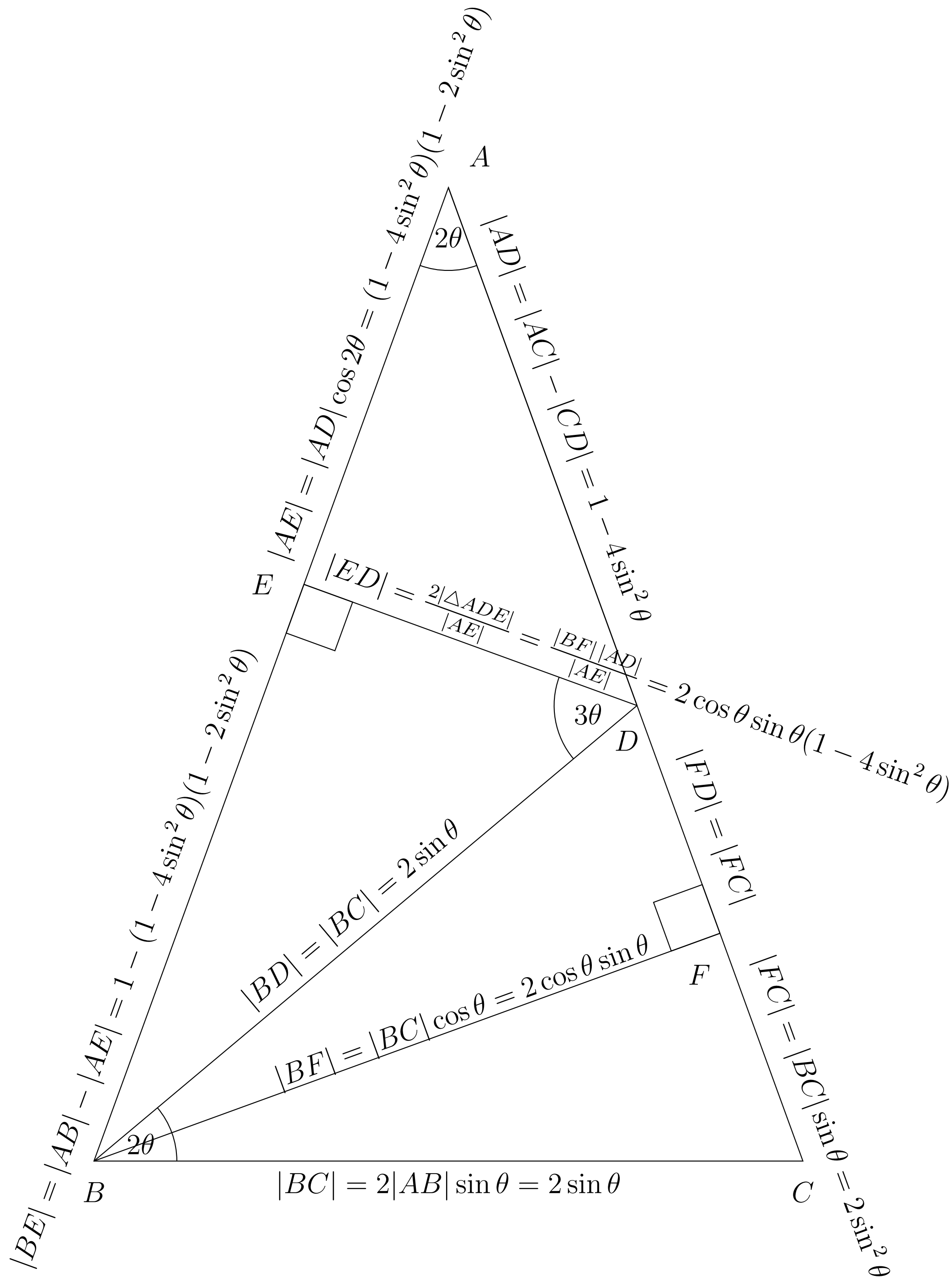

Figure 1: Triple-angle formulae. Here, $|AB| = |AC| = 1$, and $|\triangle ADE|$ is the area of $\triangle ADE$. Moreover, the identity $\cos 2\theta = \frac{|AF|}{|AB|} = 1 - 2\sin^2\theta$ is used.