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Problems 1801–1810

Parabola would like to thank Ying Wang for contributing problem Q1802 and Thomas Britz for contributing problems Q1808, Q1809 and Q1810.

Q1801 A snobbish travel writer states of an island resort, "As soon as I stepped off the plane, I lowered the median age by 40 years". Is this credible?

You will need to know that the *median* of a list of data is the middle number, if they are arranged in increasing order: for example, the median of $3, 1, 4, 1, 5, 9, 2$ is 3. If the number of numbers is even, then we take the median to be the average of the middle two: for example, the median of $6, 5, 3, 5, 8, 9$ is 5.5.

Q1802 Prove that the equation $\cos(\sin x) = \sin(\cos x)$ has no real solution.

Q1803 Let x, y, z be positive real numbers, not equal to 1, such that $xyz = 1$.

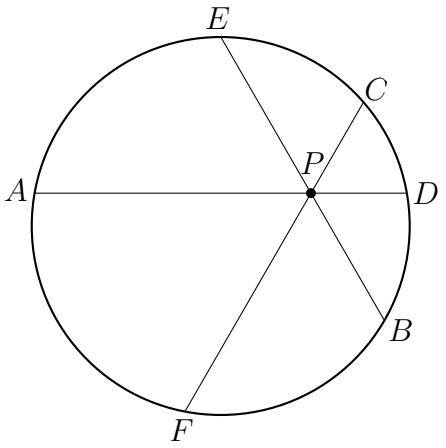
(a) Show that

$$\frac{x^2}{(x-1)^2} + \frac{y^2}{(y-1)^2} + \frac{z^2}{(z-1)^2} \geq 1\,. \qquad (*)$$

(b) Show that if the 1 on the right–hand side of $(*)$ is increased, then the statement is no longer true. That is, if c is any real number greater than 1, then there exist positive numbers $x, y, z \neq 1$ such that $xyz = 1$ and

$$\frac{x^2}{(x-1)^2} + \frac{y^2}{(y-1)^2} + \frac{z^2}{(z-1)^2} < c\,.$$

Q1804 In the picture below, the three chords AD, BE and CF intersect in a common point P at 120° angles to each other.



Prove that $|PA| + |PB| + |PC| = |PD| + |PE| + |PF|$.

Q1805 Show that the exact power of 2 dividing $n!$ is $2^{n-s(n)}$, where $s(n)$ is the sum of the binary digits of n.

Q1806 Let n be an integer. Prove that $(n-1)(n-2)(n^2-3n+12)$ is a multiple of 24.

Q1807 Let $N = 1^1 + 2^2 + \cdots + 2026^{2026}$. Is N a perfect square?

Q1808 There are five statements below.

> Exactly one or two of these five statements are true.
>
> Exactly two or three of these five statements are true.
>
> Exactly three or four of these five statements are true.
>
> Exactly four or five of these five statements are true.
>
> Exactly five or one of these five statements are true.

What are the possible numbers of true statements above?

Q1809 There are six statements below:
two green (G), two red (R) and two blue (B).

(G) Exactly two of the four red and blue statements are true.

(G) The two blue statements have the same truth value.

(R) Exactly one green statement is true.

(R) The two green statements have the same truth value.

(B) Exactly one red statement is true.

(B) The two red statements have the same truth value.

Which of these statements are true?

Q1810 As in **Q1809**, there are six coloured statements below:
two green (G), two red (R) and two blue (B).

(G) No red statement is true.

(G) Exactly one blue statement is true.

(R) No blue statement is true.

(R) Exactly one green statement is true.

(B) No green statement is true.

(B) Exactly one red statement is true.

Which of these statements are true?